MODELING OF THE ELECTRON DISTRIBUTION FUNCTION IN THE PRESENCE OF NON-LINEAR WAVE-PARTICLE INTERACTION *

Richard Kamendje[1], Sergei V. Kasilov[2], Winfried Kernbichler[1], Martin F. Heyn[1]

[1]Institut für Theoretische Physik, Technische Universität Graz
Petersgasse 16, A–8010 Graz, Austria

[2]Institute of Plasma Physics
National Science Center "Kharkov Institute of Physics and Technology"
ul. Akademicheskaya 1, 61108 Kharkov, Ukraine

1. INTRODUCTION

Electron cyclotron resonance heating (ECRH) and electron cyclotron current drive (ECCD) are widely used in experiments on modern fusion devices (tokamaks, stellarators). In particular, one of important scenarii is heating using the 2^{nd} harmonic electron cyclotron resonance for the extraordinary mode (X-mode) launched perpendicularly with respect to the main magnetic field. The applicability of the linear theory of wave absorption and, respectively, of the quasi-linear theory of the evolution of the distribution function is violated for this scenario in typical experimental conditions. In particular, the quasi-linear description of wave-particle interaction remains valid only for particles with relatively large parallel velocities such that particle flight time through the radiation beam, τ_f, is small compared to the oscillation period of particles trapped in the wave, τ_{bE}.

On the other hand, the existing adiabatic theory of non-linear wave-particle interaction which has been extensively developed [1,2,3] is also applicable in a limited region in phase space (small parallel velocities, $\tau_f \gg \tau_{bE}$). Moreover, calculations of the absorbed power within this theory were performed without taking into account the combined effect of wave-particle interaction and collision processes on the formation of the electron distribution function (see also [4]). Therefore, for typical experimental conditions this theory does not give quantitative numerical results for both the particle distribution function and the absorbed power. Therefore, a general non-linear model of wave-particle interaction, which should cover the whole phase space should be developed and combined with the modeling of Coulomb collision processes to compute numerically the particle distribution function and the power absorption.

*This work has been carried out within the Association EURATOM-ÖAW and under contract P13495-TPH with the Austrian Science Foundation.

2. FORMULATION OF THE PROBLEM

In the present work, we consider a simplified geometry with uniform main magnetic field directed along the Z-axis and a narrow Gaussian radiation beam propagating across the main magnetic field along the X-axis (see Fig.1). The system is periodic in Z-direction with a period L. E.g., such a simple model geometry can be used to describe the paraxial region of a tokamak. For the purpose of numerical computation of the distribution function, we introduce cuts at positions $z = const$ within the period L along the Z-axis (vertical lines in Fig.1). The number of cuts is chosen so that the characteristic flight time of electrons between cuts, τ_b, is small compared to the collision time, τ_c. Two of these cuts, a and b, are placed at the sides of the beam so that the relation $\delta L_b \ll \delta L \ll L$ is held. Here δL_b and δL are the width of the beam and the distance between the cuts a and b, respectively. Since particle orbits are only slightly modified by collisions during particle transitions between neighboring cuts, the kinetic equation can be solved approximately in the region between these cuts. As a result, the integral relations between particle flux densities through the neighboring cuts, $\Gamma = v_\parallel J f$, are obtained where J and f are the phase space Jacobian and the distribution function, respectively.

Figure 1: Problem geometry.

According to the presented geometry, wave-particle interaction takes place in the narrow inner region in between the cuts a and b, where the rf beam is launched. Since the width of this region, δL, is small, collisions have a negligible effect on particle orbits as compared to the effect of the wave-particle interaction. On the other hand, in the outer region between cuts b and a the amplitude of the rf field is small. Therefore, one can neglect the wave-particle interaction there.

2.1 Wave-particle interaction

The electric field distribution of the X-mode is considered in the following form,

$$\mathbf{E} = \mathbf{e}_y E \cos(kx - \omega t) e^{-\alpha z^2/2}. \quad (1)$$

Here E, k, ω and $\alpha \sim \delta L_b^{-2}$ are wave amplitude, wave vector, wave frequency and beam width parameter, respectively. The electrons are considered to be slightly relativistic so that the only relativistic effect to be retained is the dependence of the gyrofrequency on particle energy, $\omega_c \approx \omega_{c0}(1 - v^2/(2c^2))$, where ω_{c0} is the cyclotron frequency of the electrons at rest. For the same reason, the finite Larmor radius effects are taken into account only in the leading order over the small parameter $kv_\perp/\omega_c \sim v/c \ll 1$. Using the Hamiltonian formalism (see, e.g., Refs. [1,2,3]) and retaining in the Hamiltonian only the term corresponding to the second harmonic resonance, $\omega \approx 2\omega_c$, the equations of electron motion can be presented in the dimension-less form [4],

$$\frac{\mathrm{d}w}{\mathrm{d}\tau} = \varepsilon\, w\, e^{-\tau^2} \sin\psi, \quad (2)$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\tau} = \delta - w + \varepsilon\, e^{-\tau^2} \cos\psi, \quad (3)$$

where $\tau = \sqrt{\alpha/2}|v_\parallel| t$ defines the dimensionless time, ψ the wave-particle phase, w the perpendicular particle energy,

$$w = \sqrt{\frac{2}{\alpha}} \frac{|\omega_{c0}| v_\perp^2}{|v_\parallel| c^2}, \quad (4)$$

$$\varepsilon = \sqrt{\frac{2}{\alpha}} \frac{N|\omega_{c0}|}{|v_\parallel|} \frac{E}{B_0}, \quad (5)$$

$$\delta = \sqrt{\frac{2}{\alpha}} \frac{|\omega_{c0}|}{|v_\parallel|} \left(2 - \frac{\omega}{|\omega_{c0}|} - \frac{v_\parallel^2}{c^2}\right). \quad (6)$$

Here N, B_0, c and $v_\perp$ are wave refraction index, main magnetic field, speed of light and perpendicular velocity, respectively. In this approximation, the parallel velocity $v_\parallel$ stays constant during the interaction.

Considering solely the wave-particle interaction, the kinetic equation reduces to the Vlasov equation which can be exactly solved with the help of the method of characteristics. This solution gives a relation between the flux densities of particles entering the rf-beam through the cut a ($v_\parallel > 0$) and particles leaving the beam through the cut b. Taking into account the fact that the wave-particle phase is randomized between successive passes of a particle through the wave beam due to collisions, the particle flux density on the cut b is then related to the particle flux density on the cut a by the relation

$$\Gamma_b(w) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}w_0 \int_{-\pi}^{\pi} \mathrm{d}\psi' \delta(w - W(w_0, \psi'))\Gamma_a(w_0). \quad (7)$$

Here $W(w_0, \psi')$ and $\Psi(w_0, \psi')$ are the values of w and ψ at the cut b given by the solution of the equations of motion with initial conditions w_0 and ψ' at the previous intersection with the cut a.

2.2 Collisional interaction

In the outer region, a stationary kinetic equation neglecting the wave-particle interaction is considered,

$$v_\parallel \frac{\partial f}{\partial z} = \hat{L}_c f, \quad (8)$$

where $\hat{L}_c$ is the collision operator. A simplified collision model including only the diffusion over perpendicular energy is used,

$$\hat{L}_c f = \frac{\partial}{\partial w} D_c \left(\frac{\partial f}{\partial w} + \frac{f}{w_T}\right), \quad (9)$$

where w_T is given by (4) with $v_\perp = v_T$, v_T is the thermal velocity, $D_c = \nu_c w_T^2$ and ν_c is the collision frequency. Since the parallel velocity is not changed within this model, the problem is one dimensional. The method of Laplace transform provides an exact solution of equation (8), which is presented as an integral expression for the particle phase space flux density on the cut a through the same quantity on the cut b and a Green's function G.

$$\Gamma_a(w) = \int_0^\infty dw' \Gamma_b(w') G(w, w'). \quad (10)$$

In the case of weak collisions, $\tau_b \ll \tau_c$, the Green's function has an approximate form,

$$G(w, w') = \frac{1}{\sqrt{4\pi D_c \tau_b}} \exp\left(\frac{\left(w - w' + D_c \tau_b w_T^{-1}\right)^2}{4 D_c \tau_b}\right). \quad (11)$$

In a two dimensional treatment which is required for realistic collision model, a similar mapping relation can be found in the limit of weak collisions [5]. Combining relations (7) and (10) one can eliminate Γ_b (or Γ_a) and obtain an integral equation for Γ_a (or Γ_b) which has to be solved numerically.

3. SOLVING THE MAPPING EQUATION

The integral form of the kinetic equation is of the type of a Fredholm second kind integral equation

which can be solved with standard Monte Carlo methods. The statistical estimate for the solution is achieved by introducing a test particle position w. A Markov chain is then built as sequence of particle positions: $w_1^{(a)} \to w_1^{(b)} \to w_2^{(a)} \to w_2^{(b)} \ldots \to w_k^{(a)} \to w_k^{(b)} \to w_{k+1}^{(a)} \to w_{k+1}^{(b)} \to \ldots$
Recurrence relations are given through,

$$w_k^{(b)} = W\left(w_k^{(a)}, \psi\right), \tag{12}$$

$$w_{k+1}^{(a)} = w_k^{(b)} + \sqrt{2D_c\tau_b}\xi_g - \frac{D_c\tau_b}{w_T}, \tag{13}$$

where ξ_g is a random number with a normal distribution. The estimate of the solution is obtained as

$$\Gamma^{(e)}(w) = \left\langle \frac{1}{N}\sum_{k=1}^{N} \delta\left(w - w_k\right)\right\rangle, \quad (N \to \infty). \tag{14}$$

Here $\langle \ldots \rangle$ denotes the box average defined by

$$\langle a(w)\rangle = \frac{1}{\Delta w}\int\limits_{i\Delta w}^{(i+1)\Delta w} \mathrm{d}w' a(w') \tag{15}$$

for w from the interval $i\Delta w < w < (i+1)\Delta w$ where i is an integer and $\Delta w \ll w_T$.

4. RESULTS AND DISCUSSION

Parameters for typical medium size toroidal magnetic trap have been chosen, $L = 2\pi R$, $R = 200$ cm, $\alpha = 0.25$ cm^{-2}, $B_0 = 20$ kG, with electron temperature T_e=3 keV and an input power of the beam P_{ECRH}=400 kW. Our computation shows that a non-linear plateau-like structure is formed on the distribution function around the resonance zone in velocity space. Due to the combined effects of non-linear wave-particle interaction and collisions, the derivative of the distribution function with respect to the perpendicular velocity may locally become positive. This effect becomes more pronounced with an increased collision frequency. The existence of the region with positive derivative can be shown analytically if one considers the wave-particle interaction in the adiabatic limit. In this case the relation (7) is simplified, $\Gamma_b(w) = \Gamma_a(w)$, for $|w_0 - \delta| > \Delta w_{max}$, and $\Gamma_b(w) = \frac{1}{2}\left(\Gamma_a(w) - \Gamma_a(2\delta - w)\right)$ for $|w_0 - \delta| < \Delta w_{max}$. Thus $\Gamma_b(w)$ is symmetric around δ in the region of the nonlinearly broadened resonance zone $|w_0 - \delta| < \Delta w_{max}$. Therefore $\Gamma_b(w)$ has a positive derivative, unless the distribution of incoming electrons, Γ_a, is constant with respect to w in this region. This is not the case in the presence of collisions.

The positive derivative of the distribution function indicates that non-linear effects of ECRH may cause the electron Bernstein wave instability. The bursts of the non-thermal electron cyclotron emission which can be attributed to such an instability have been observed in high power ECRH experiments on the stellarator W-7AS [6].

Figure 2: distribution function $f(w)$ as a function of perpendicular energy obtained using the general non-linear model (A and B) and the adiabatic model (C and D) for the densities $n = 10^{12}cm^{-3}$ (A and C) and $n = 10^{14}cm^{-3}$ (B and D).

REFERENCES

[1] I. A. Kotel'nikov and G. V Stupakov. *J. Plasma Physics*, **45**, 19 (1991).

[2] I.A. Kotel'nikov and G.V. Stupakov. *Phys. Fluids B*, **2**, 881 (1990).

[3] D. Farina, R. Pozzoli, and M. Romé. *Phys. Fluids B*, **3**, 3065 (1991).

[4] M. F. Heyn, S. V. Kasilov, W. Kernbichler, H. Maassberg, M. Romé, U. Gasparino, and N. Marushchenko. In *26th EPS Conf. on contr. Fusion and Plasma Physics*, volume 23J, page 1625, Maastricht, 14-18 june 1999. The European Physical Society, Petit-Lancy.

[5] S. V. Kasilov, V. E. Moiseenko, and M. F. Heyn. *Physics of Plasmas*, **4**, 2422 (1997).

[6] M. Romé, V. Erckmann, U. Gasparino, H. J. Hartfuss, G. Kühner, H. Maassberg, and N. Marushchenko. *Plasma Phys. Contr. Fusion*, **39**, 117 (1997).