Temporal evolution of drift Alfvén waves and instabilities in an inhomogeneous plasma with homogeneous shear flow

Vladimir S. Mikhailenko* and Vladimir V. Mikhailenko
Kharkiv National University, Kharkiv-108, 310108, Ukraine

Martin F. Heyn
Institut für Theoretische Physik, Technische Universität Graz, Petersgasse 16, A–8010 Graz, Austria

Swadesh M. Mahajan
Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78701
and Abdus Salam International Centre for Theoretical Physics, Trieste, Italy
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The temporal evolution of drift Alfvén waves in an inhomogeneous plasma of low and finite pressure with homogeneous shear flow is studied as an initial value problem without the use of spectral expansion in time. The cases of plasma with cold and hot ions, weak and strong flow shear are considered separately. It is shown that the conventional modal structure of the stable and unstable drift and Alfvén waves holds only for a limited time in the initial stage of its evolution. For larger times, nonmodal effects due to the velocity shear define the development of drift Alfvén waves and drift Alfvén instabilities. For the regimes of low flow shear, which corresponds to the period of the low-to-high transition, the long time evolution of these instabilities as well as their saturation are determined by the nonlinear effects such as the nonlinear decorrelation effect. In contrast, the plasma with strong flow shear, which corresponds to the regime of the developed transport barriers, is stable against the development hydrodynamic drift Alfvén and resistive drift Alfvén instabilities. The frequency increase caused by the shear flow brings the Alfvén wave phase speed close to the electron thermal speed where strong electron Landau damping occurs. At this stage, a kinetic approach for the description of these waves becomes necessary.

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I. INTRODUCTION

It is well recognized now that the short-wavelength drift-type waves are the source of anomalous transport in toroidal confinement systems. There has been substantional theoretical progress in understanding the structure and stability of these waves in a sheared magnetic field. The conventional theory of short-scale drift toroidal modes is based on the ''ballooning transform'' [1], perhaps, the most effective method for calculating the spectrum and global structure of drift-ballooning modes [2].

The experimental discovery of the transition from the low-confinement state to the high-confinement state, in which the suppression of turbulence and reduction of anomalous transport was detected, open a new page in the theory of drift wave and drift turbulence in toroidal confinement systems. Experiments have shown that, together with the transition to the improved confinement state (H-mode regime), a tokamak plasma develops large variations in the radial electric field, and strong poloidal plasma shear flows [3–6]. The large variations are also limited to the same small edge layer in which the velocity shear length L_v is found to be much less than the magnetic shear length L_B. In fact, the shearing rate in this region is of the order of, or even larger than, the typical drift wave frequency. It stands to reason, then, that the nature and evolution of the low frequency waves and instabilities in the edge layer will be different from that in the plasma core, where the magnetic shear is the primary determinant of the spatial structure and temporal evolution of these waves. Unfortunately, the ballooning transform method ceases to be useful for problems that involve significant shear flow, and may be suitable only in finding the spectrum (and growth rates) in the limit of vanishing velocity shear [7–10]. Other methods have to be developed for the analysis of a plasma with strong flow shear. In Ref. [11] a new approach to the theory of drift waves in plasma with strongly sheared flows ($L_v \ll L_B$) was proposed. The character of fluctuations for such a system is dominated by the sheared flow, and minor effects of magnetic shear can be safely omitted. In Ref. [11] the effects of the shear flow with a constant shearing rate were worked out for low frequency driftlike perturbations in a collisional (Hasegawa-Wakatani model in a slab) as well as a collisionless (Hasegawa-Mima model) plasma [11]. It was shown that the shear flow has not only a stabilizing effect on the resistive drift instability, but also leads to principal changes in the structure of the basic drift wave. Consequently the structural elements of the drift turbulence and, thus, the correlation properties of the turbulence are bound to change. The solution of the relevant initial value problem [11] also reveals strong nonmodal features like time dependent frequencies and wave numbers and nonseparable spatiotemporal behavior of the perturbations [12–17]. The standard form for the modal structure of drift perturbations $\exp(ik_y v_{de} t)$, with v_{de} being the electron diamagnetic drift velocity, survives in a shear flow for a limited time only. With increasing time, this wave ultimately transforms into a convective cell with zero frequency and an amplitude decay-

*Electronic mail address: mikhailenko@pem.kharkov.ua

ing as $1/t^2$. It was also shown in Ref. [11], that the radial wave number of the drift wave grows secularly with time in shear flows (see also Refs. [8,9]). Implying the onset of the associated nonlinear effects, this drives initially minor effect of the magnetic shear into greater insignificance. This approach proves to be useful also in the studies of temporal evolution of electrostatic toroidal ion temperature gradient driven instability [20] and Rayleigh-Tailor instability [21] in plasma with shear flow.

In our present paper we extend these studies by including the electromagnetic waves and instabilities. We undertake, what may be called, a nonmodal (in time) investigation of the drift Alfvén wave in the presence of a sheared velocity field. The drift Alfvén wave is considered to be an important component of the low frequency plasma turbulence which may be responsible for the anomalous transport in toroidal confinement devices. Its structure and stability properties in the strongly sheared edge plasma of a tokamak will, perhaps, bear crucially on our understanding of the nature of the *H*-mode. Since the edge layer is rather narrow, we employ a slab model to study both a low ($m_e/m_i \gg \beta$) and finite ($m_e/m_i \ll \beta \ll 1$) pressure plasma, where $\beta = 4\pi P/B^2$ is the ratio of thermal to magnetic pressure. Driven by the experiment, we also assume the velocity shear length L_v to be much less than the magnetic shear length L_B. In fact, the confining magnetic field will be considered to be shearless and homogeneous. The temporal evolution of the spatial Fourier modes of perturbations is studied directly as an initial value problem; the standard spectral expansion is not expected to reveal essential features of the temporal evolution. In Sec. II the basic equations governing the temporal evolution of drift Alfvén waves in a plasma with homogeneous shear flow (constant shearing rate) are derived. In Secs. III and IV, the initial value problem is approximately solved to chart out the time history of the mode: in the former the plasma has cold ions, $T_i \ll T_e$, while in the latter the electron and ion temperatures are comparable, $T_i \leqslant T_e$, where T_i and T_e are the respective the ion and electron temperatures.

II. BASIC EQUATIONS

The governing equations of the present model are the equations for the longitudinal motion of electrons, for the quasineutrality of the current density, and for the perturbations of the electron pressure $\tilde{p}_e$ and ion pressure $\tilde{p}_i$. In the drift approximation, the system of equations reduces to the following system for the parallel component of the perturbed magnetic potential $\tilde{A}_\parallel$, the electrostatic potential $\tilde{\phi}$, and for $\tilde{p}_e$ and $\tilde{p}_i$ [22],

$$\frac{d}{dt}\left(\tilde{A}_\parallel - \frac{c^2}{\omega_{pe}^2}\nabla_\perp^2 \tilde{A}_\parallel\right) + v_{de}\frac{\partial \tilde{A}_\parallel}{\partial y} = -c\frac{\partial\tilde{\phi}}{\partial z} + \frac{c}{en_{e0}}\frac{\partial \tilde{p}_e}{\partial z} + \frac{c}{en_{e0}B_0}\boldsymbol{\nabla}\tilde{A}_\parallel \cdot [\mathbf{b}_0 \times \boldsymbol{\nabla}\tilde{p}_e], \tag{1}$$

$$\frac{d\tilde{p}_e}{dt} + en_{e0}v_{de}\frac{\partial\tilde{\phi}}{\partial y} + \frac{c\Gamma_e T_{e0}}{4\pi e}\left(\frac{\partial}{\partial z} + \frac{1}{B_0}[\boldsymbol{\nabla}\tilde{A}_\parallel \times \mathbf{b}_0]\cdot\boldsymbol{\nabla}\right) \times \nabla_\perp^2 \tilde{A}_\parallel = 0, \tag{2}$$

$$\frac{d\tilde{p}_i}{dt} - en_{i0}v_{di}\frac{\partial\tilde{\phi}}{\partial y} = 0, \tag{3}$$

$$\left(\frac{d}{dt} + v_{di}\frac{\partial}{\partial y}\right)\nabla_\perp^2\tilde{\phi} = -\frac{c}{en_{i0}B_0}\nabla_\perp([\mathbf{b}_0 \times \boldsymbol{\nabla}\tilde{p}_i]\cdot\boldsymbol{\nabla})\nabla_\perp\tilde{\phi} - \nabla_\perp(\mathbf{v}_{i\parallel}\cdot\boldsymbol{\nabla})\nabla_\perp\tilde{\phi} - \frac{v_A^2}{c}\left(\frac{\partial}{\partial z} + \frac{1}{B_0}[\boldsymbol{\nabla}\tilde{A}_\parallel \times \mathbf{b}_0]\cdot\boldsymbol{\nabla}\right)\nabla_\perp^2\tilde{A}_\parallel, \tag{4}$$

with ω_{pe} being the electron plasma frequency, v_A being the Alfvén velocity, $\Gamma = 5/3$ being the adiabatic constant and $v_{de(di)} = \mp c/(en_{e0}B_0)dP_{e0(i0)}/dx$ being the electron (ion) diamagnetic velocity. $P_{e0(i0)}(x)$ is the inhomogeneous equilibrium electron (ion) pressure. The operator d/dt in Eqs. (1)–(4) is defined for a shear flow transverse to the magnetic field $\mathbf{B}_0 = B_0\mathbf{b}_0$ (directed along the z axis), with uniform shearing rates, i.e, $\mathbf{v}_0(x) = v_0' x \mathbf{e}_y$, where v_0' is independent of x, as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_0' x\frac{\partial}{\partial y} + \mathbf{v}_E \cdot \boldsymbol{\nabla} \tag{5}$$

with $\mathbf{v}_E = (c/B_0)[\mathbf{b}_0 \times \boldsymbol{\nabla}\tilde{\phi}]$.

The conventional way to resolve this system of equations or their particular forms is to assume the perturbed quantities vary as $(A_\parallel, \phi, p_e, p_i) \sim [A_\parallel(x), \phi(x), p_e(x), p_i(x)]\exp(ik_y y + ik_z z - i\omega t)$. It is known, however, [see, for example, Ref. [24])] that for the case of shear flow the solutions of such modal form do not give the complete solution to the problem and more rigorous in this case will be the solution of the initial value problem for the system considered. The application of the direct and inverse Laplace transform in time formally complete the solution of the initial value problem [24]. The Laplace inversion integrals, however, appear to be involved for the computation of time evolution for finite time values. As a rule this method gives analytical results only for an asymptotically large times. These asymptotic results may become useless in the comparative analysis of the relative importance of nonlinear and linear effects in the evolution in time of the plasma instability in shear flow, because all temporal processes in time evolution, which in some definite time may be more important than the modal or the nonlinear ones, are omitted. Therefore it is unlikely to receive on this way analytical solutions valid for any finite time and to analyze with them the temporal evolution of drift Alfvén waves or stabilization of the drift Alfvén instabilities.

Here we have used another approach, which gives easy and transparent treating of the problem considered. It is known that with such homogeneous shear flows, the solution of the initial value problem is greatly facilitated by a transformation to coordinates which are convected with the sheared flow. Such a transformation is given by [11] (see also Refs. [12–19])

$$\tau = t, \quad \xi = x, \quad \eta = y - v_0' x t, \quad \zeta = z. \tag{6}$$

In the new coordinates the linearized system (1)–(4) for the nondimensional variables $\phi = e\tilde{\phi}/T_e$, $A_\parallel = eA_\parallel/eT_e$, $p_e = \tilde{p}_e/n_{0e}T_e$, $p_i = \tilde{p}_i/n_{0e}T_e$ becomes

$$\frac{\partial}{\partial \tau}\left(A_\parallel - \frac{c^2}{\omega_{pe}^2}\nabla_\perp^2 A_\parallel\right) + v_{de}\frac{\partial A_\parallel}{\partial \eta} = -c\frac{\partial \phi}{\partial \zeta} + c\frac{\partial p_e}{\partial \zeta}, \tag{7}$$

$$\frac{\partial p_e}{\partial \tau} + v_{de}\frac{\partial \phi}{\partial \eta} + \Gamma\frac{c^2}{\omega_{pe}^2}\frac{v_{Te}^2}{c}\frac{\partial}{\partial \zeta}\nabla_\perp^2 A_\parallel = 0, \tag{8}$$

$$\frac{\partial p_i}{\partial \tau} - v_{di}\frac{\partial \phi}{\partial \eta} = 0, \tag{9}$$

$$\left(\frac{\partial}{\partial \tau} + v_{di}\frac{\partial}{\partial \eta}\right)\nabla_\perp^2 \phi = v_0'\frac{\partial}{\partial \eta}\left(\frac{\partial p_i}{\partial \xi} - v_0'\tau\frac{\partial p_i}{\partial \eta}\right) - \frac{v_A^2}{c}\frac{\partial}{\partial \zeta}\nabla_\perp^2 A_\parallel, \tag{10}$$

where the operator $\nabla_\perp^2$ is

$$\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \eta^2} + \left(\frac{\partial}{\partial \xi} - v_0'\tau\frac{\partial}{\partial \eta}\right)^2. \tag{11}$$

The new coordinates were designed to transfer the spatial dependence to the time domain. The preceding spatially homogenous system may be Fourier analyzed in the new spatial variables ξ, η, ζ as

$$A_\parallel(\tau, k_\perp, l, k_z) = \int\int\int d\xi d\eta d\zeta A_\parallel(\tau, \xi, \eta, \zeta) \times \exp\{-ik_\perp\xi - il\eta - ik_z\zeta\}, \tag{12}$$

taking into account the condition that $k_\perp L_{pe,i} \gg 1$, where $L_{pe,i}$ is the scale length of the inhomogeneous equilibrium electron (ion) pressure, to obtain the following system for the evolution of the spatial Fourier harmonics $A_\parallel$, ϕ, p_e, and p_i,

$$\frac{\partial}{\partial T}\left[\left(1 + \frac{c^2 l^2}{\omega_{pe}^2}(1+T^2)\right)A_\parallel\right] + iC_e A_\parallel = -i\frac{c}{v_A}S\phi + i\frac{c}{v_A}Sp_e, \tag{13}$$

$$\frac{\partial p_e}{\partial T} + iC_e\phi = i\Gamma\frac{c}{v_A}\frac{v_{Te}^2}{\omega_{pe}^2}Sl^2(1+T^2)A_\parallel, \tag{14}$$

$$\frac{\partial p_i}{\partial T} = iC_i\phi, \tag{15}$$

$$\frac{\partial((1+T^2)\phi)}{\partial T} + iC_i(1+T^2)\phi = -Tp_i - i\frac{v_A}{c}S(1+T^2)A_\parallel. \tag{16}$$

In systems (13)–(16), $\rho_s = v_s/\omega_{ci}$ is the ion-sound Larmor radius, $v_s = (\Gamma_e T_e/m_i)^{1/2}$ is the ion-sound velocity, v_{Te} is the electron thermal velocity, and v_A is the Alfvén velocity. The dimensionless time T and the dimensionless parameters S, C_e, and C_i are defined as

$$T = v_0'\tau - \frac{k_\perp}{l}, \quad S = \frac{k_z v_A}{v_0'}, \quad C_{e,i} = \frac{l v_{de,di}}{v_0'}. \tag{17}$$

Systems (13)–(16) together with the Fourier transformed initial data $A_\parallel(0,k_\perp,l,k_z)$, $p_e(0,k_\perp,l,k_z)$, $p_i(0,k_\perp,l,k_z)$, and $\phi(0,k_\perp,l,k_z)$ constitutes the general initial value problem. In the following, the temporal evolution of the drift Alfvén waves in a plasma with cold ions ($T_i \ll T_e$) and with hot ions ($T_i \leqslant T_e$) in the case of "weak" flow shear, when $S \gg 1$, is considered.

III. TEMPORAL EVOLUTION OF ALFVÉN WAVES IN A PLASMA WITH COLD IONS

This section is devoted to a study of Alfvén waves in a homogeneous as well as an inhomogeneous (in local approximation) plasma with cold ions, $T_i \to 0$, for a broad range of electron pressure with $\beta = 4\pi P/B^2$ (the ratio of thermal to magnetic pressure) spanning the ranges $\beta \ll m_e/m_i$ and $1 \gg \beta \gg m_e/m_i$.

A. Temporal evolution of Alfvén waves in a homogeneous low pressure plasma ($\beta \ll m_e/m_i$)

For $\beta \ll m_e/m_e$, i.e., with both $P_{i0} \to 0$ and $P_{e0} \to 0$, the system of equations (13)–(16) reduces to

$$\frac{d^2((1+T^2)\phi)}{dT^2} + S^2\frac{v_{Te}^2}{v_A^2}\frac{1}{l^2\rho_s^2}\phi = 0. \tag{18}$$

The solution to Eq. (18) has a modal form with frequency $\Omega = k_\parallel v_{Te}/l\rho_s$ for $T \ll 1$. For larger times $T \gg 1$, it may be approximately solved for the electrostatic and magnetic potentials,

$$\phi \approx \frac{1}{T^{3/2}}(C_1 T^{i\omega_1} + C_2 T^{-i\omega_1}), \tag{19}$$

$$A_\parallel(T) \approx \frac{ic}{k_\parallel v_A^2}\frac{1}{T^{5/2}}\left(\frac{C_1}{\frac{1}{2}+i\omega_1}T^{i\omega_1} + \frac{C_2}{\frac{1}{2}-i\omega_1}T^{-i\omega_1}\right), \tag{20}$$

where $\omega_1 = \sqrt{\Omega^2/(v_0')^2 - 1/4}$. It follows from Eqs. (19) and (20) that the flow shear has fundamentally altered the modal time behavior from the conventional to a powerlike time dependence of the spatial Fourier harmonic of the perturbed potentials. An algebraic decay is imposed with the magnetic potential decaying more rapidly with time than the electrostatic potential. This feature—the different time dependence of different perturbations in a linear system, is strictly a nonmodal element introduced by the velocity shear.

B. Temporal evolution of Alfvén waves in a finite pressure plasma ($\beta \gg m_e/m_i$) with cold ions

For cold ions, $T_i \to 0$, but for the finite electron pressure, the appropriately approximated system (13)–(16) may be combined into a single third order differential equation for $\phi(\tau, k_\perp, l, k_z)$,

$$\frac{\partial^2}{\partial T^2}\left\{\left(\frac{1}{1+T^2}+\frac{c^2 l^2}{\omega_{pe}^2}\right)\frac{\partial}{\partial T}[(1+T^2)\phi]\right\} + S^2\frac{\partial}{\partial T}[(1+l^2\rho_s^2(1+T^2))\phi] + iC_e S^2\phi + iC_e\frac{\partial}{\partial T}\left\{\frac{1}{1+T^2}\frac{\partial}{\partial T}[(1+T^2)\phi]\right\}=0. \quad (21)$$

Equation (21), together with the initial conditions obtained with Eqs. (13)–(16),

$$\phi(T,k_\perp,l,k_z)|_{T=-k_\perp/l}=\phi(0), \quad (22)$$

$$\left.\frac{\partial\phi}{\partial T}\right|_{T=-k_\perp/l} = k_z v_A\left[\frac{2}{S}\frac{lk_\perp}{k_\perp^2+l^2}\phi(0) - i\frac{v_A}{c}A_\parallel(0)\right], \quad (23)$$

$$\left.\frac{\partial^2\phi}{\partial T^2}\right|_{T=-k_\perp/l} = k_z^2 v_A^2 p_e(0) + k_z^2 v_A^2 \times\left[\frac{8}{S^2}\frac{(lk_\perp)^2}{(k_\perp^2+l^2)^2} - \frac{2}{S^2}\frac{l^2}{l^2+k_\perp^2} - 1\right]\phi(0) - \frac{1}{S}\frac{v_A}{c}k_z^2 v_A^2\left[C + 2i\frac{lk_\perp}{k_\perp^2+l^2}\right]A_\parallel(0), \quad (24)$$

constitutes the initial value problem for the linear drift and Alfvén waves in a plasma shear flow with cold ions. In the limit of weak flow shear, i.e., $S \gg 1$, the initial value problem may be formally solved in the eikonal approximation

$$\phi(T) = \frac{1}{1+T^2}\exp\left(S\int_{-k_\perp/l}^{T} \mathrm{d}T' f(T')\right), \quad (25)$$

where

$$f(T) = f_0(T) + \frac{1}{S}f_1(T) + \frac{1}{S^2}f_2(T) + \cdots. \quad (26)$$

From this general solution we will first extract the waveforms pertinent to the homogeneous plasma ($C_e = 0$), and then the drift waves in an inhomogeneous plasma ($C_e \neq 0$).

If $C_e = 0$ (homogeneous plasma), the mode equation (21) may be integrated once to obtain the inhomogeneous differential equation

$$\frac{\partial}{\partial T}\left\{\left(\frac{1}{1+T^2}+\frac{c^2 l^2}{\omega_{pe}^2}\right)\frac{\partial}{\partial T}[(1+T^2)\phi]\right\} + S^2\{[1+l^2\rho_s^2(1+T^2)]\}\phi = S^2[\rho_s^2(l^2+k_\perp^2)\phi(0) + p_e(0)]. \quad (27)$$

Notice that the integrability implies the existence of a constant of the motion which lowers the effective dimensionality of the system; in this from three to two. For $S^2 \gg 1$, the asymptotic solution for the homogeneous equation (27) could be easily written in terms of elliptic integrals. We shall, however, avoid that rather opaque representation, and write approximate but simpler forms relevant to different but well-defined intervals of the normalized time T. We find that Alfvén waves in their ''classical'' modal form (with thermal corrections),

$$\phi_{1,2}(\tau) \approx \exp\{\pm i k_\parallel v_A \tau[1 + (k_\perp^2 + l^2)\rho_s^2]^{1/2}\}, \quad (28)$$

will exist only within the time interval $0 < \tau < (v_0')^{-1}(|T| < 1)$, i.e., for physical times less than the inverse of the shearing rate. In the next well-defined time interval

$$1 \ll T \ll (l\rho_s)^{-1}, \quad (29)$$

the Alfvén waves begin to acquire the nonmodal slow-decay imposed on a modal form,

$$\phi_{1,2} \sim \frac{1}{T}\exp(\pm iST). \quad (30)$$

This mixture of the modal and nonmodal behavior continues in the interval

$$(l\rho_s)^{-1} \ll T \ll (l\rho_s)^{-1}\frac{v_{Te}}{v_A}, \quad (31)$$

in which the solution

$$\phi_{1,2} \sim T^{-3/2}\exp\left(\pm i\frac{S}{2}l\rho_s T^2\right) \quad (32)$$

is characterized by a ''frequency'' increasing linearly with time. Finally, for asymptotic times (for times larger than the various ''times'' in the problem),

$$T \gg (l\rho_s)^{-1}\frac{v_{Te}}{v_A}, \quad (33)$$

the solution

$$\phi_{1,2} \sim T^{-2}\exp\left(\pm iST\frac{v_{Te}}{v_A}\right) \quad (34)$$

corresponds to an algebraically decaying but oscillating mode. In the ultimate stage, the wave that began as an ordinary Alfvén wave ends up propagating with the electron thermal speed v_{Te}. This metamorphosis is due to the finite electron mass effect which is usually omitted in the study of Alfvén waves in a plasma with finite pressure, $\beta > m_e/m_i$. Through the agency of the shear which transfers energy between different wave numbers (in particular induces upward cascading), this normally neglected term becomes dominant for large enough times when the effective wave numbers become large. Since the waves propagating with the the electron thermal speed will be subject to strong electron Landau damping, the flow shear may become a rather effective mechanism for damping the Alfvén or drift Alfvén waves in the shear layer of the high-confinement tokamak discharges; the shear connects the wave (which may be growing) to the highly damped part of the spectrum causing it to ultimately decay. Needless to say that for these conditions a kinetic description for the Alfvén wave evolution becomes necessary.

In the space-time coordinates ξ, η, t, solutions (28)–(34) have the form

$$\phi_{1,2}(t,\xi,\eta,z)=\frac{1}{(2\pi)^3}\int dk_\perp\int dl\int dk_z\Phi_{1,2}(t,k_\perp,l,k_z)\times e^{ik\xi+il\eta+ik_z z}e^{i\Gamma_\pm(t,k,l,k_z)}, \quad (35)$$

where $\Phi_{1,2}(t,k_\perp,l,k_z)$ contains the initial data, and the nonexponential dependence of these solutions. For a wave packet peaked about a central wave number $K_0=(k_{\perp 0},l_0,k_{z0})$ the components of the group velocity in the comoving frame are equal to [11]

$$v_{gx}=-\frac{\partial}{\partial t}\left(\frac{\partial\Gamma_\pm}{\partial k_\perp}\right)_{k_{\perp 0},l_0,k_{z0}},$$

$$v_{gy}=-\frac{\partial}{\partial t}\left(\frac{\partial\Gamma_\pm}{\partial l}\right)_{k_{\perp 0},l_0,k_{z0}},\quad v_{gz}=-\frac{\partial}{\partial t}\left(\frac{\partial\Gamma_\pm}{\partial k_z}\right)_{k_{\perp 0},l_0,k_{z0}}. \quad (36)$$

It follows from the solution that the "classical" pattern for the propagation of an Alfvén wave packet with the group velocity

$$v_{gx}=\mp v_A\frac{k_{0\parallel}}{k_{0\perp}}\frac{k_{0\perp}^2\rho_s^2}{(1+(k_{0\perp}^2+l_0^2)\rho_s^2)^{1/2}},$$

$$v_{gy}=\mp v_A\frac{k_{0\parallel}}{l_0}\frac{l_0^2\rho_s^2}{(1+(k_{0\perp}^2+l_0^2)\rho_s^2)^{1/2}},$$

$$v_{gz}=\mp v_A(1+(k_{0\perp}^2+l_0^2)\rho_s^2)^{1/2}, \quad (37)$$

will be found only within times $0<\tau<(v_0')^{-1}$(or $|T|<1$). In the time interval (29), $v_{gz}=v_A$ and $v_{gx}=v_{gy}=0$, i.e., the wave packet does not propagate across the magnetic field. For times (31), we have

$$v_{gx}=\pm v_A k_{0\parallel}\rho_s,\quad v_{gy}=\mp v_A\frac{k_{0\perp}}{l_0}k_{0\parallel}\rho_s,$$

$$v_{gz}=\mp v_A l_0\rho_s\left(v_0't-\frac{k_{0\perp}}{l_0}\right), \quad (38)$$

revealing that in this interval, the component of the group velocity directed along the velocity inhomogeneity, v_{gx}, is in a direction opposite to the initial (37), i.e., after a period of blocking, the packet is reflected in x. In times (33) we have ultimately $v_{gx}=v_{gy}=0$ and $v_{gz}=v_{Te}$; the packet propagates strictly along the magnetic field.

The particular solution of inhomogeneous equation (27),

$$\phi_*=\frac{\rho^2(k_\perp^2+l^2)\phi(0)+p_e(0)}{1+\rho_s^2[l^2+(k_\perp-v_E'l\tau)^2]} \quad (39)$$

is of the type obtained for a convective cell or a vortex [11,25,23]; it is also the solution for the ultimate stage of the evolution of the drift wave [11]. Taking into account the initial conditions (22) and (23), we write down the complete solution for the potential ϕ,

$$\phi(\tau)=C_1\phi_1(\tau)+C_2\phi_2(\tau)+\phi_*(\tau), \quad (40)$$

where the constants $C_{1,2}$ are calculated to be

$$C_{1,2}=\left(1+\frac{k_\perp^2}{l^2}\right)^{-1/2}[1+\rho_s^2(l^2+k_\perp^2)]^{-1/4}\left[\frac{\phi(0)-p_e(0)}{[1+\rho_s^2(l^2+k_\perp^2)]^2}\mp i\frac{v_A}{c}A_\parallel(0)\pm\frac{1}{S}\left(\frac{lk_\perp}{l^2+k_\perp^2}\phi(0)-\frac{lk_\perp\rho_s^2[\rho_s^2(l^2+k_\perp^2)\phi(0)+p_e(0)]}{[1+\rho_s^2(l^2+k_\perp^2)]^2}\right)\right]. \quad (41)$$

In deriving $C_{1,2}$, small terms of the order $(m_e/m_i)(k_\perp^2+l^2)\rho_s^2/\beta$ have been omitted. Thus, in a homogeneous plasma the perturbation spectrum consists of two Alfvén waves, determined by Eqs. (28)–(34), and a convective cell (39). The temporal evolution of the electron pressure perturbation p_e is obtained from Eqs. (13) and (16),

$$p_e(\tau)=-\rho_s^2[(l^2+(k_\perp-v_0'l\tau)^2]\phi(\tau)+\rho_s^2(l^2+k_\perp^2)\phi(0)+p_e(0), \quad (42)$$

which grows as T in the time interval (29), like $T^{1/2}$ in the time interval (31), and stays constant for larger times. From Eqs. (39) and (42), we may also learn that the electron pressure perturbation connected with the convective cell, $\phi_*(\tau)$, may be no larger than $O(S^{-2})$.

From Eq. (16), we find that the magnetic potential $A_\parallel$ associated with the Alfvén waves is given by

$$A_{\parallel(1,2)}=-\frac{c}{v_A}\phi_{(1,2)}(T)\left(\frac{1+l^2\rho_s^2(1+T^2)}{\left(1+\frac{v_A^2}{v_{Te}^2}l^2\rho_s^2(1+T^2)\right)}\right)^{1/2}+O(S), \quad (43)$$

which, for the successive intervals enumerated earlier, becomes in Eq. (29),

$$A_{\parallel(1,2)}\sim-\frac{1}{T}\exp(\pm iST), \quad (44)$$

in Eq. (31)

$$A_{\parallel(1,2)}\sim-\frac{1}{T^{1/2}}\exp\left(\pm i\frac{S}{2}l\rho_s T^2\right), \quad (45)$$

and in interval (33)

$$A_{\|(1,2)}\sim-\frac{1}{T^2}\exp\left(\pm iST\frac{v_{Te}}{v_A}\right). \qquad (46)$$

For the convective cell, the magnetic potential is

$$A_{\|*}(T)=2i\frac{c}{v_A}\frac{\rho_s^2 l^2}{S}T\frac{\rho_s^2(k_\perp^2+l^2)\phi(0)+p_e(0)}{[1+l^2\rho_s^2(1+T^2)]^2}. \qquad (47)$$

It grows as T when $T<(l\rho_s)^{-1}$, and decays strongly as T^{-3} for $T>(l\rho_s)^{-1}$. It is interesting to note that in this case of finite pressure plasma, as well as in the case of low pressure plasma, the velocity shear introduces different nonmodal time dependence for different perturbations.

The reader may note that the asymptotic solutions of Eq. (27) for extremely strong shear, $S\ll 1$, are readily obtained by the transformation of Eq. (27) to the "slow" time variable $T_1=ST$. With T_1 as the time variable, Eq. (27) becomes

$$\frac{\partial}{\partial T_1}\left\{\left(\frac{S^2}{T_1^2}+\frac{l^2c^2}{\omega_{pe}^2}\right)\frac{\partial}{\partial T_1}\left[\left(1+\frac{T_1^2}{S^2}\right)\phi(T_1)\right]\right\} +\left[1+\rho_s^2 l^2\left(1+\frac{T_1^2}{S^2}\right)\right]\phi(T_1) =\rho_s^2(k_\perp^2+l^2)\phi(0)+p_e(0). \qquad (48)$$

For T_1 finite and $S\to 0$, i.e., for $T>S^{-1}$, one obtains from Eq. (48) solutions (34) for Alfvén waves, and solution (39) for the convective cell.

C. Inhomogeneous plasma with cold ions

In this section, asymptotic solutions for weak shear ($S\gg 1$) of Eq. (21) are derived separately for the weak ($S\gg C\sim 1$), and strong ($S\sim C\gg 1$) plasma inhomogeneities.

1. Weak plasma inhomogeneity

After substitution of Eq. (25) with Eq. (26) into Eq. (21), the three solutions of the inhomogeneous plamsa may be written as

$$\phi_{1,2}(T)=\frac{1}{(1+T^2)^{1/2}(1+l^2\rho_s^2(1+T^2))^{1/4}\left(1+\frac{c^2l^2}{\omega_{pe}^2}(1+T^2)\right)^{1/4}}\exp\left\{\pm iS\int_{-k_\perp/l}^{T}dT'\left(\frac{1+l^2\rho_s^2(1+T'^2)}{1+\frac{c^2l^2}{\omega_{pe}^2}(1+T'^2)}\right)^{1/2}\right.$$
$$\left.+\frac{iC}{2}\left[\frac{1}{l\rho_s\sqrt{1+l^2\rho_s^2}}\tan^{-1}\left(\frac{l\rho_sT'}{\sqrt{1+l^2\rho_s^2}}\right)-\frac{1}{\sqrt{l\rho_s\frac{v_A^2}{v_{Te}^2}}\sqrt{1+\frac{c^2l^2}{\omega_{pe}^2}}}\tan^{-1}\left(\frac{\frac{cl}{\omega_{pe}}T'}{\sqrt{1+\frac{c^2l^2}{\omega_{pe}^2}}}\right)\right]\right\}, \qquad (49)$$

and

$$\phi_3(T)=\frac{1+\rho_s^2(l^2+k_\perp^2)}{1+l^2\rho_s^2(1+T^2)}\times\exp\left\{-i\frac{C}{l\rho_s\sqrt{1+l^2\rho_s^2}}\left[\tan^{-1}\left(\frac{l\rho_sT}{\sqrt{1+l^2\rho_s^2}}\right)+\tan^{-1}\left(\frac{k_\perp\rho_s}{\sqrt{1+l^2\rho_s^2}}\right)\right]\right\}, \qquad (50)$$

with Eq. (49) describing the Alfvén wave modified by the plasma inhomogeneity, and Eq. (50) representing the electron drift wave in the presence of the flow [11]. From Eq. (50) (see also Ref. [11]) one observes that because of the shear flow, the drift wave is ultimately transformed into a convective cell. It is interesting to note that in Eqs. (49) and (50) the terms in the exponentials connected with the plasma inhomogeneity are proportional to the $\tan^{-1}$ function. Thus, the dependence on plasma density inhomogeneity becomes negligible after some time, and asymptotically the Alfvén waves in the weakly inhomogeneous plasma will have the same form as their homogeneous counterpart (40) with somewhat modified coefficients [determined by initial conditions (22)–(24)]. The quenching of the inhomogeniety effects is due to the shear-induced increase in the effective wave numbers which renders the weakly inhomogeneos plasma virtually homogenous for the high k perturbations.

2. Strong plasma inhomogeneity

In this case, the function $f_0(T)$ in Eq. (26) is found from the equation

$$(f_0^2+1)(f_0+i\delta)=-(1+T^2)l^2\rho_s^2 f_0\left(1+\frac{c^2}{\omega_{pe}^2\rho_s^2}f_0^2(T)\right), \qquad (51)$$

where $\delta = C/S$. In the limit $v_0' = 0$ and $m_e = 0$, Eq. (51) reduces to the well-known dispersion equation coupling the drift and Alfvén waves in an inhomogeneous plasma without shear flow (see, e.g., Ref. [26]). In the presence of a shear flow, however, Eq. (51) contains an important time dependence which leads to temporal changes in the structure of the plasma perturbations. Equation (51) has no simple solutions and this is also true for the much more complicated equation for f_1, the next function in the eikonal expansion.

For $|T| \ll 1$, the solutions of Eq. (51) are similar to that obtained for a plasma without shear flows. But for larger time T, the shear flow will modify the solutions substantially. Writing Eq. (51) in the form

$$f_0\left[f_0^2 + i\delta f_0 + 1 + l^2\rho_s^2(1+T^2) + \frac{c^2 l^2}{\omega_{pe}^2}(1+T^2) f_0^2(T)\right] = -i\delta, \tag{52}$$

one finds for $T \gg 1$, the first solution

$$f_{01} \approx \frac{-i\delta}{1 + l^2\rho_s^2(1+T^2)}, \tag{53}$$

which corresponds to the drift wave. For time interval (31), the equation

$$f_0^2 + i\delta f_0 + l^2\rho_s^2 T^2 = 0 \tag{54}$$

may be approximately solved for the two solutions

$$f_{02,03} \approx -\frac{i}{2}\left(\delta \pm \sqrt{\delta^2 + 4l^2\rho_s^2 T^2}\right). \tag{55}$$

These solutions are the Alfvén waves modified by the plasma inhomogeneity. For even larger times T, Eq. (33), the approximate solutions of Eq. (51) are

$$f_{02,03} \approx -\frac{i\delta\omega_{pe}^2}{2T^2c^2l^2} + i\frac{v_{Te}}{v_A}\left(1 - \frac{\omega_{pe}^2}{2T^2c^2l^2}\right), \tag{56}$$

and one concludes that for strong plasma inhomogeneities, the drift wave perturbation as well as the effects of the inhomogeneity on the Alfvén waves disappear with time. In the long time limit, the temporal evolution of the initial disturbance will be governed by an equation similar to Eq. (40), and therefore the drift Alfvén wave structure would have transformed into a convective cell and a strongly damped (due to electron Landau damping) Alfvén wave.

IV. TEMPORAL EVOLUTION OF DRIFT ALFVÉN INSTABILITIES IN A PLASMA WITH HOT IONS

Now we consider plasma with hot ions $T_i \leqslant T_e$. It is well known that in shearless case inhomogeneous plasma with hot ions is unstable against the hydrodynamic drift Alfvén instability [26]. Here we consider the effect of flow shear on the temporal evolution of that instability in the regime of weak flow shear, which corresponds to the stage of the period of the low-to-high (L-H) transition, and in the regime of strong flow shear, which corresponds to the stage of the developed transport barriers. For that goal we have to consider full system of equations (13)–(16). It is suitable now to present this system of equations into matrix form and to introduce new variables U and Ψ, by the relations

$$U = (1+T^2)\phi, \quad \Psi = \left(1 + \frac{c^2l^2}{\omega_{pe}^2}(1+T^2)\right)A_\parallel. \tag{57}$$

We consider the regimes of weak flow shear, for which $S \gg 1$, $S/C_e = O(1)$, and $S/C_i = O(1)$ and strong flow shear, for which $S \ll 1$, $S/C_e = O(1)$ and $S/C_i = O(1)$, separately. Introducing a parameter λ by the formal change $S \to \lambda S$, $C_e \to \lambda C_e$, $C_i \to \lambda C_i$, Eqs. (13)–(16) may be rewritten formally as

$$\frac{dq}{dT} + F(T,\lambda)q = 0, \tag{58}$$

where $q = (\Psi, p_e, p_i, U)$ is a column 4-vector and the matrix $F(T,\lambda)$ is equal to

$$F(\lambda,t) = \begin{pmatrix} \dfrac{i\lambda C_e}{1 + \dfrac{c^2l^2}{\omega_{pe}^2}(1+T^2)} & -i\lambda S\dfrac{c}{v_A} & 0 & i\lambda S\dfrac{c}{v_A}\dfrac{1}{1+T^2} \\ -i\dfrac{c}{v_A}\dfrac{v_{Te}^2}{\omega_{pe}^2}S\lambda\dfrac{l^2(1+T^2)}{1 + \dfrac{c^2l^2}{\omega_{pe}^2}(1+T^2)} & 0 & 0 & i\lambda C_e\dfrac{1}{1+T^2} \\ 0 & 0 & 0 & -i\lambda C_i\dfrac{1}{1+T^2} \\ i\dfrac{v_A}{c}\lambda S\dfrac{1+T^2}{1 + \dfrac{c^2l^2}{\omega_{pe}^2}(1+T^2)} & 0 & T & i\lambda C_i \end{pmatrix}.$$

Clearly, the regimes of weak and strong flow shear correspond to large and small values of the parameter λ, respectively.

Solution of Eq. (58) in the case of weak flow shear is looked for in the WKB form

$$q(T,\lambda)=a(T)\exp\left(\lambda\int f(T,\lambda)\mathrm{d}T\right), \tag{59}$$

where $a(T)$ is a column vector, and $f(T,\lambda)=\Sigma_{i=0}^{\infty}f_i(T)\lambda^{-i}$. For $f_0(T)$ one can obtain one root $f_0(T)=0$, and other three roots are determined from the equation

$$(f_0^2(T)+iC_if_0(T)+S^2)(f_0(T)+iC_e) = -\left[f_0^2(T)\frac{m_e}{m_i}\frac{1}{\beta}+S^2\right](1+T^2)l^2\rho_s^2(f_0(T)+iC_i), \tag{60}$$

where the identity $c^2l^2/\omega_{pe}^2=(m_e/m_i)(1/\beta)l^2\rho_s^2$ was used. For $l\rho_s\ll 1$ in times $T\ll 1$ the right-hand side of Eq. (60) is small. Omitting the right term we have for $T\ll 1$ solutions

$$f_{01,02}=i\frac{|C_i|}{2}\pm\left(\frac{C_i^2}{4}+S^2\right)^{1/2}, \tag{61}$$

which define two Alfvén waves and solution

$$f_{03}=-iC_e, \tag{62}$$

which defines the electron drift wave. The accounting for the right term gives the hydrodynamic drift Alfvén instability. The maximal growth rate will be for oscillations in the vicinity of the crossing of the solutions f_{01} and f_{03} and is equal to (for $T_i=T_e$) [26]

$$\gamma=\sqrt{\frac{2}{3}}\, lv_{de}l\rho_s(1+T^2)^{1/2}\left(\frac{m_e}{m_i}\frac{1}{\beta}-2\right)^{1/2}. \tag{63}$$

In times $T\geqslant 1$ the time dependence of the growth rate becomes important. Because for time $t=(v_0')^{-1}$ we have

$$\frac{\gamma}{v_0'}=\left(\frac{lv_{de}}{v_0'}\right)l\rho_s\sqrt{\frac{m_e}{m_i\beta}}, \tag{64}$$

the long waves with $l\rho_s<(v_0'/lv_{de})(m_i\beta/m_e)$ in times less than the inverse modal growth rate begin to grow with nonmodal growth rate (63) which is linearly growing with time. It is obvious that for these waves the conventional estimate for the suppression of the instability due to the nonlinear decorrelation effect [5], $\gamma\sim v_0'$, with modal growth rate γ, have to be modified.

In times $T>T_1=(1/l\rho_s)(m_i\beta/m_e)^{1/2}$, the right term in Eq. (60) is no longer small and conventional procedure, which gives the drift Alfvén instability with growth rate (63) is no more valid. For these times it is suitable to rewrite Eq. (60) in the form

$$\left[f_0^2(T)+\frac{S^2l^2\rho_s^2(1+T^2)}{1+\frac{m_e}{m_i}\frac{1}{\beta}l^2\rho_s^2(1+T^2)}\right][f_0(T)+iC_i] = -\frac{S^2[f_0(T)+iC_e]+iC_ef_0(T)[f_0(T)+iC_i]}{1+\frac{m_e}{m_i}\frac{1}{\beta}l^2\rho_s^2(1+T^2)}. \tag{65}$$

The right term in Eq. (65) may be considered for large times, $T\gg 1$, as a small. Without the right term we have three uncoupled solutions two of which

$$f_{01,02}=\pm iSl\rho_s\left(\frac{1+T^2}{1+\frac{m_e}{m_i}\frac{1}{\beta}l^2\rho_s^2(1+T^2)}\right)^{1/2} \tag{66}$$

may be attributed to shear flow transformed Alfvén waves, and the third solution

$$f_{03}(T)=-iC_i \tag{67}$$

is associated with the electron drift wave transformed by the shear flow. We remind the reader that for $T_i=0$, solution (67) corresponds to the zero frequency convective cell [11]. With crossing condition $f_{01}=f_{03}$ of wave brunches, which in shearless case gives growth rate (63), we find for $f_0=f_{01}+\delta f_0$ that

$$\delta f_0=\pm i\frac{S(C_e+|C_i|)}{\sqrt{2l\rho_s}(1+T^2)^{1/4}\left(1+\frac{m_e}{m_i}\frac{1}{\beta}l^2\rho_s^2(1+T^2)\right)^{1/4}}. \tag{68}$$

One can see that in this stage of evolution, there is no drift Alfvén instability. This result show that in times $T>T_1$ shear flow leads to the changing the frequency of the waves and as a result to the violating their coupling. However, this result is mathematical artifact, because for times $T>T_1$ we have [27]

$$\gamma(t)t>\frac{lv_{de}}{v_0'}\frac{1}{l\rho_s}\left(\frac{m_e}{m_i\beta}-2\right)^{1/2}\gg 1, \tag{69}$$

and prior to the development this effect the nonlinear effects similar to nonlinear decorrelation effect will suppress this nonmodal instability.

It is followed from Eq. (63) that the hydrodynamic drift Alfvén instability is excited in low β plasmas with $\beta<m_e/2m_i$. In higher β plasma the resistive drift Alfvén instability [26] is possible. That instability is the electromagnetic counterpart of the electrostatic dissipative drift instability, temporal evolution of which in plasama with shear flow was considered in Ref. [11]. The relevant system of equations for this instability will be the systems (13)–(16) with new term $\nu_{ei}(c^2l^2/\omega_{pe}^2)(1+T^2)A_\parallel$ added to the left-hand side of Eq. (13). This term is responsible for the collisional

dissipation of longitudinal motion of electrons. With the accounting for this term the equation for $f_0(T)$ takes the form

$$(f_0^2(T)+iC_if_0(T)+S^2)(f_0(T)+iC_e) = -\left[f_0^2(T)\frac{m_e}{m_i}\frac{1}{\beta}+f_0(T)R_e\frac{m_e}{m_i}\frac{1}{\beta}+S^2\right] \times(1+T^2)l^2\rho_s^2(f_0(T)+iC_i), \tag{70}$$

where dimensionless parameter $R_e=\nu_{ei}/v_0'\rightarrow\lambda R_e$ was used. For $l\rho_s\ll 1$ in times $T\leqslant 1/l\rho_s$ the right-hand side of Eq. (70) is small. The maximal growth rate for resistive drift Alfvén instability will be for oscillations in the vicinity of the crossing of the solutions f_{01} and f_{03} determined by Eqs. (61) and (62). This growth rate is equal to

$$\gamma\approx\frac{1}{\sqrt{6}}\nu_{ei}l\rho_s\frac{m_e}{m_i\beta}\frac{(1+T^2)^{1/2}}{\left(2-\frac{m_e}{m_i\beta}\right)^{1/2}} \tag{71}$$

in the case of low collisionality ($\nu_{ei}\ll l v_{de}$) and

$$\gamma\approx\frac{1}{\sqrt{3}}l\left(\rho_s\nu_{ei}l v_{de}\frac{m_e}{m_i\beta}\right)^{1/2}(1+T^2)^{1/2} \tag{72}$$

in the case of strong collisionality ($\nu_{ei}\gg l v_{de}$). As in the case of the hydrodynamic drift Alfvén instability, in times $T\gg 1$ the time dependence of growth rates (71) and (72) becomes important. It results in the nonmodal evolution of waves. The waves with $l\rho_s<(v_0'/\nu_{ei})(m_i\beta/m_e)$ in the low collision case and waves with $l\rho_s<[v_0'/(\nu_{ei}l v_{de})^{1/2}](m_i\beta/m_e)$ in the case of strong collisions will grow in time with linearly growing with time growth rate (63). It is obvious that for these waves the conventional estimate for the suppression of the instability due to the nonlinear decorrelation effect [5] $\gamma\sim v_0'$, with modal growth rate γ also must be modified to account for the nonmodal effects.

For times $T\geqslant 1/l\rho_s$ the right-hand sides in Eq. (70) are not a small and above procedure is no more applicable. For that times it is more suitable to present Eq. (70) in the form

$$\left[f_0^2(T)+\frac{l^2\rho_s^2(1+T^2)\left(S^2+f_0(T)R_e\frac{m_e}{m_i\beta}\right)}{1+\frac{m_e}{m_i}\frac{1}{\beta}l^2\rho_s^2(1+T^2)}\right](f_0(T)+iC_i) = -\frac{S^2(f_0(T)+iC_e)+iC_ef_0(T)(f_0(T)+iC_i)}{1+\frac{m_e}{m_i}\frac{1}{\beta}l^2\rho_s^2(1+T^2)}. \tag{73}$$

Omitting the right-hand side term in Eq. (73) for large times, $T\gg 1$, as a small we have three uncoupled solutions two of which,

$$f_{01,02}(T)=-\frac{R_e}{2}\pm\left(\frac{R_e^2}{4}-S^2\frac{m_i\beta}{m_e}\right)^{1/2}, \tag{74}$$

correspond to the linearly damped solutions and the third solution $f_{03}(T)$ is defined by Eq. (62). It is obvious that on these times the resistive drift Alfvén instability is absent. However, the times $T\geqslant 1/l\rho_s$ appear to be much more longer than the inverse growth rate time. For that time we have estimates

$$\gamma(t)t>\frac{\nu_{ei}}{v_0'}\frac{1}{l\rho_s}\frac{m_e}{m_i\beta}\frac{1}{\left(2-\frac{m_e}{m_i\beta}\right)^{1/2}}\gg 1 \tag{75}$$

in low collisional case and

$$\gamma(t)t>\frac{(\nu_{ei}l v_{de})^{1/2}}{v_0'l\rho_s}\left(\frac{m_e}{m_i\beta}\right)^{1/2}\gg 1 \tag{76}$$

in strong collisional case.

The above analysis shows that nonlinear decorrelation effect is responsible for the suppression of both drift Alfvén instabilities in the period of the L-H transition.

For the solution of systems (13)–(16) in the regime of strong flow shear which corresponds to the stage of the developed transport barriers [5], it is suitable to introduce new dimensionless time $T_1=\lambda T$. With new time the system of equations (58) with accounted collisional dissipation takes the form

$$\frac{\partial\Psi}{\partial T_1}+iC_e\frac{\Psi(T_1)}{1+\frac{c^2l^2}{\omega_{pe}^2}\left(1+\frac{T_1^2}{\lambda^2}\right)}+\frac{c^2l^2}{\omega_{pe}^2}R_e\frac{\left(1+\frac{T_1^2}{\lambda^2}\right)\Psi(T_1)}{1+\frac{c^2l^2}{\omega_{pe}^2}\left(1+\frac{T_1^2}{\lambda^2}\right)}+i\frac{c}{v_A}S\frac{U(T_1)}{1+\frac{T_1^2}{\lambda^2}}-i\frac{c}{v_A}Sp_e=0, \tag{77}$$

$$\frac{\partial p_e}{\partial T_1}+iC_e\frac{U(T_1)}{1+\frac{T_1^2}{\lambda^2}}-i\frac{v_{Te}^2}{cv_A}\frac{c^2l^2}{\omega_{pe}^2}S\frac{\left(1+\frac{T_1^2}{\lambda^2}\right)\Psi(T_1)}{1+\frac{c^2l^2}{\omega_{pe}^2}\left(1+\frac{T_1^2}{\lambda^2}\right)}=0, \tag{78}$$

$$\frac{\partial p_i}{\partial T_1} - iC_i \frac{U(T_1)}{1+\frac{T_1^2}{\lambda^2}} = 0, \tag{79}$$

$$\frac{\partial U}{\partial T_1} + iC_i U(T_1) + i\frac{v_A}{c} S \frac{(1+T_1^2)\Psi(T_1)}{1+\frac{c^2 l^2}{\omega_{pe}^2}\left(1+\frac{T_1^2}{\lambda^2}\right)} + \frac{T_1}{\lambda} p_i = 0. \tag{80}$$

In time interval $\lambda\omega_{pe}/cl \gg T_1 \gg \lambda$ with the accuracy to the terms of the order of $O(\lambda^2)$ the solutions of system (77)–(80) are

$$A_{\|}(\tau) \approx \exp\left(-\frac{c^2 l^2}{\omega_{pe}^2}\frac{\nu_{ei}}{3v_0'}\left(v_0'\tau - \frac{k}{l}\right)^3\right), \tag{81}$$

$$p_e(\tau) \approx -i\frac{v_{Te}^2}{cv_A}\frac{k_{\|}v_A}{\nu_{ei}}\exp\left(-\frac{c^2 l^2}{\omega_{pe}^2}\frac{\nu_{ei}}{3v_0'}\left(v_0'\tau - \frac{k}{l}\right)^3\right), \tag{82}$$

$$p_i(\tau) \approx \text{const}, \tag{83}$$

$$\phi(\tau) \approx i\frac{v_A}{c}\frac{k_{\|}v_A}{\nu_{ei}}\exp\left(-\frac{c^2 l^2}{\omega_{pe}^2}\frac{\nu_{ei}}{3v_0'}\left(v_0'\tau - \frac{k}{l}\right)^3\right). \tag{84}$$

It follows from these nonmodal solutions that any disturbances in the time interval $\lambda\omega_{pe}/cl \gg T_1 \gg \lambda$ are stable against the resistive drift Alfvén instability in the presence of flow with strong shear. This instability is absent also for times $T_1 \gg \lambda\omega_{pe}/lc$, where solutions of system (77)–(80) [with omitted terms of the order of $O(\lambda^2)$] are

$$A_{\|} \sim \frac{\exp\left(-\frac{\nu_{ei}\tau}{2}\right)}{\left(v_0'\tau - \frac{k}{l}\right)^2}(D_1\cos\delta\tau + D_2\sin\delta\tau), \tag{85}$$

$$p_e \sim \exp\left(-\frac{\nu_{ei}\tau}{2}\right)(F_1\cos\delta\tau + F_2\sin\delta\tau), \tag{86}$$

$$p_i \sim \text{const}, \tag{87}$$

$$\phi \sim \frac{1}{\left(v_0'\tau - \frac{k}{l}\right)^2}(G_1\cos lv_{di}\tau + G_2\sin lv_{di}\tau). \tag{88}$$

These solutions show that on the ultimate stage perturbations of pressures $p_{i,e}$ and potentials ϕ, $A_{\|}$ are stable and have different time dependencies.

The governing system of the equations for collisionless plasma in the case of strong shear flow is determined by Eqs. (77)–(80) with $R_e=0$. In time period $\lambda \ll T_1 \ll \lambda\omega_{pe}/cl$, we obtain the following solutions for perturbed potentials and pressures

$$[A_{\|}(\tau), p_e(\tau)] \sim \frac{1}{\left(v_0'\tau - \frac{k_\perp}{l}\right)^{1/2}}\exp\left[\pm i\frac{l\rho_s}{2}k_{\|}v_A \times\left(v_0'\tau - \frac{k_\perp}{l}\right)^2\right], \tag{89}$$

$$p_i \approx \text{const}, \tag{90}$$

$$\phi(\tau) \sim \frac{1}{\left(v_0'\tau - \frac{k_\perp}{l}\right)^{3/2}}\exp\left[\pm i\frac{l\rho_s}{2}k_{\|}v_A\left(v_0'\tau - \frac{k_\perp}{l}\right)^2\right], \tag{91}$$

where the terms of the order of $O(\lambda^2)$ are omitted. Finally, for times $T_1 > \lambda\omega_{pe}/lc$ we find, that on the ultimate stage the evolution of perturbed potentials and electron pressure perturbation is similar with their evolution in the case $p_i=0$,

$$A_{\|} \sim \frac{1}{\tau^2}\sin(k_{\|}v_{Te}\tau), \tag{92}$$

$$p_e \sim \sin(k_{\|}v_{Te}\tau), \tag{93}$$

$$p_i \approx \text{const}, \tag{94}$$

$$\phi \sim \frac{1}{\tau^2}[D\sin(lv_{de}\tau) + E\sin(k_{\|}v_{Te}\tau)]. \tag{95}$$

The stabilized perturbations of the magnetic potential and the electron pressure will ultimately be transformed, as in the case of cold ions, into waves whose phase speed is the electron thermal speed v_{Te} rather than the Alfvén speed v_A. These waves damp due to electron Landau damping and only the perturbation of the electrostatic potential with the frequency lv_{di} will survive which, however, also damps slowly like τ^{-2}.

V. CONCLUSIONS

An analytical theory of the temporal evolution of drift Alfvén waves in an inhomogeneous plasma of low and finite pressure with homogeneous shear flow is presented. The cases of plasma with cold and hot ions, weak and strong flow shear are considered separately. It is shown that the conventional modal structure of the stable and unstable drift and Alfvén waves holds only for a limited time in the initial stage. For larger times, these waves acquire a more complicated nonmodal structure with time dependent frequencies and amplitudes and with different time dependence for perturbation of potentials $A_\parallel$, ϕ and perturbation of pressures p_i, p_e.

(1) We obtain that in low pressure plasma $\beta \ll m_e/m_e$ with cold ions the flow shear has fundamentally altered the modal time behavior from the conventional to a powerlike time dependence of the spatial Fourier harmonic of the perturbed potentials. An algebraic decay is imposed with the magnetic potential decaying more rapidly with time than the electrostatic potential. This feature—the different time dependence of different perturbations in a linear system, is a strictly nonmodal element introduced by the velocity shear.

(2) We find that for a plasma with finite pressure $(m_e/m_i \ll \beta \ll 1)$ and cold ions, the velocity shear reduces the effects of other inhomogeneities (pressure) on Alfvén waves, and leads to an asymptotic behavior $1/t^2$ for the amplitudes of both the electrostatic and magnetic potentials. In time, the shear flow also raises the Alfvén wave frequency—ultimately imparting it a phase speed (along the magnetic field) approaching the electron thermal speed. Such a wave suffers strong electron Landau damping necessitating a kinetic approach for a proper long time description. This shear-induced temporal transformation (a joint effect of the shear flow and the finite electron mass) to a highly damped mode will provide a mechanism for the damping of perturbations which could, indeed, be growing without the flow shear.

(3) In plasma with hot ions hydrodynamic drift Alfvén and resistive drift Alfvén instabilities may be developed. For the regimes of low flow shear, which corresponds to the period of the low-to-high transition [3–5], the nonmodal effects may actually control the temporal evolution of instabilities on time scales less than their inverse growth rates. However, the long time evolution of these instabilities as well as their saturation are determined by the nonlinear effects such as the nonlinear decorrelation effect. In contrast, the plasma with strong flow shear, which corresponds to the regime of the developed transport barriers [5], is stable against the development hydrodynamic drift Alfvén and resistive drift Alfvén instabilities. For plasma with hot ions $(T_i \ll T_e)$, the frequency of the electron drift wave ultimately reduces to the frequency of the ion drift wave, lv_{di}. The drift-wave transformation into a convective cell with zero frequency and an amplitude decaying as $1/t^2$ seems to be an inherent property of the temporal evolution of the drift mode in a plasma with cold ions $(T_i = 0)$ [11].

It is interesting to note that the analysis with longitudinal homogeneous shear flow with $\mathbf{v}_0(x) = v_0' x \mathbf{b}_0$, where v_0' is independent of x, leads to equations identical to Eqs. (13)–(16), if one redefines the time T and the parameters S and $C_{e,i}$ as

$$T = v_0' \frac{k_z}{l} \tau - \frac{k_\perp}{l}, \quad S = \frac{k_z v_A}{v_0' \frac{k_z}{l}}, \quad C_{e,i} = \frac{l v_{de,di}}{v_0' \frac{k_z}{l}}. \tag{96}$$

Therefore, the solution of the initial value problem for the longitudinal shear flow may be obtained from the results for the transversal case presented above by simply changing v_0' to $v_0' k_z/l$.

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