3D MODELING OF THE MINORITY DISTRIBUTION FUNCTION DURING RF HEATING*

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1. Introduction

Plasma heating with waves in the ion cyclotron range of frequencies (ICRF) at the fundamental resonance of the minority ions is one of the basic mechanisms for auxiliary heating in tokamaks. Due to the absorption of toroidal angular momentum by the waves, besides the neoclassical transport an additional radial transport of minority ions occurs [1]. Although it is generally difficult to separate all the effects contributing, there seems to be some recent experimental evidence favoring this type of transport [2].

Most of the energy gained by the minority ions from the rf waves is transferred by Coulomb collisions to the bulk plasma and therefore the additional transport is not important for the energy balance in large scale devices where the banana width is small compared to the minor radius of the tokamak. However, in regimes of high specific rf input power per minority particle, the effect of this additional transport on the minority ion concentration has been predicted to be comparable or even larger than the neoclassical transport [1]. In the present contribution this effect is studied using a 3D Monte Carlo (MC) code.

2. 3D Bounce Averaged Code

In the present study, the 3D bounce averaged equation is solved with a MC code for the case the velocity part of the minority distribution function behaves quasi-stationary, i.e., the time scale of transport effects is assumed to be much larger than the relaxation time scale of the minority ions in velocity space. Specifically, this is implemented by taking the small ρ_L Larmor radius limit and simultaneously the small $1/k_\|$ parallel wave length limit such that the product $\rho_L k_\|$ stays constant. This guarantees the survival of convection even in the small Larmor radius limit. In this regime, the random walks of the test particles do not produce significant displacements over the radius during the typical relaxation time in velocity space.

The bounce averaged equation underlying the present code is formulated in the variables

*This work was partly supported by the Association EURATOM-OEAW under contract number ERB 5004 CT 96 0020 and by the Ukrainian Foundation of Fundamental Research project 2.4/699.

r, the small radius, p, the momentum, and λ, the pitch on the minimum B plane. This form of the bounce averaged equation naturally follows from the stochastic mapping equation for the special case of toroidal symmetry [3]. The bounce averaged equation in this set of variables is related to the bounce averaged equation used in [4] by a tensor transformation to canonical variables, i.e., the energy, the magnetic moment, and the toroidal canonical momentum. The coefficients $\overline{D}^{ij}$ of the orbit integrated collision operator are computed with the help of the deformation tensor (Jacobian matrix) simultaneously with the the particle orbits.

The quasi-linear diffusion operator is taken in the same form as in [4]. For ICRF waves with fixed toroidal wave number the diffusion operator becomes double degenerate in phase space and, as a result, the corresponding Monte Carlo (MC) process for Δu^i, the stochastic shift in 3D velocity phase space, can be described by a single random number ξ with zero average and the variance equal to one,

$$\Delta u^i = \sqrt{2\varepsilon \overline{D}^{\varepsilon\varepsilon}}\,\xi\beta^i + \varepsilon\mathcal{F}^{\rm i}, \qquad \overline{\rm D}^{\rm ij} = \overline{\rm D}^{\varepsilon\varepsilon}\beta^{\rm i}\beta^{\rm j}, \qquad \mathcal{F}^{\rm i} = \frac{1}{\rm J}\frac{\partial}{\partial {\rm u^j}}{\rm J}\overline{\rm D}^{\rm ij},$$

where

$$\beta^r = \frac{1}{m\omega_c}\frac{1}{\hat{h}_\vartheta}\left[\frac{m\hat{h}_\varphi}{\lambda p}\left(1-\frac{B_0}{B_{\rm res}}\right) - \frac{k_\varphi}{\omega}\right], \quad \beta^p = \frac{m}{p}, \quad \beta^\lambda = \frac{m}{\lambda p^2}\left(1-\lambda^2-\frac{B_0}{B_{\rm res}}\right)$$

$$\overline{D}^{\varepsilon\varepsilon} = \frac{\pi e^2}{8\left|\frac{{\rm d}\omega_c}{{\rm d}s}\right|}\frac{v_{\perp{\rm res}}}{|v^*_{\|{\rm res}}|}|E_0^+|^2 \begin{cases} 1, & \text{fast wave (subcritical density)},\\ \dfrac{1}{\left|W\left(\frac{v_{\|{\rm res}}}{\sqrt{2}\omega_c}\right)\right|^2}, & \text{fast wave (supercritical density)}, \end{cases}$$

Here, $\overline{D}^{ij}$ is the diffusion tensor integrated along the drift orbits in local Lagrangian coordinates with J the Jacobian. $B_{\rm res}$ and B_0 are the magnetic field strengths in the resonance point and the minimum B surface, $\hat{h}_\vartheta$ and $\hat{h}_\varphi$ are the physical components of the unit vector along the magnetic field, k_φ is the toroidal component of the wave vector, $v_{\perp{\rm res}}$ and $v^*_{\|{\rm res}}$ are the perpendicular and parallel particle velocities in the cyclotron resonance point $\omega = \omega_c$, E_0^+ is the left polarized electric field amplitude at the cyclotron resonance point, and ε is the parameter for biasing the time evolution. Finite Larmor radius effects have not been taken into account. The plasma dispersion function W is used to model the screening of left polarized electric field component of the fast wave in the regime of supercritical minority heating.

3. Results

The radial convection velocity is obtained by following an ensemble of test particles in time and taking ensemble averages,

$$V_r = \left\langle \lim_{N\to\infty}\frac{\sum_{i=1}^N \Delta r_i(\mathbf{u_i})}{\sum_{\mathbf{i=1}}^{\mathbf{N}}\varepsilon(\mathbf{u_i})\tau_{\mathbf{b}}(\mathbf{u_i})}\right\rangle,$$

where Δr_i is the shift in radius during one MC step, τ_b is the particle return time to the minimum B plane (bounce time). Practically, N should be large enough so that the denominator is large compared to the collision time. In the quasi-stationary case, the same result can be obtained by phase space averaging, i.e., an ensemble of test particles is followed until the result saturates and the statistics is sufficient for a reasonable noise level. In particular, the splitting/roulette algorithm has been used which had been previously applied to model the distribution function [5]. Figure 1 shows the contours of the ion minority distribution function (a) and the distribution of the convection velocity in phase space for the case of ion minority heating with a subcritical minority density (b). As expected, the convection primarily stems from the trapped particles whose banana orbits pass through the cyclotron resonance zone. The value of Stix's parameter for the present computation is $\xi = 17$. The convection velocity satisfies the relation

$$\frac{V_r}{w_t} = -\frac{k_\varphi w}{m\omega\omega_c \hat{h}_\vartheta},$$

where w_t is the specific power absorbed per trapped resonant particle. The computed value of V_r/w where w is the specific power per resonant particle turns out to be 0.4 of the theoretical value given above and reflects the fact that the rest of the energy is absorbed by passing particles which contribute less to the convection. For the case of supercritical minority heating instead of 0.4 this factor is somewhat smaller, 0.3, because the amount of energy absorbed by trapped particles is reduced.

4. Summary

The rf induced convection has been modeled using a 3D bounce averaged Monte Carlo code. Two types of rf diffusion operators have been considered, namely the modified Stix's operator which corresponds to subcritical minority density and the operator which takes into account the polarization in the resonance zone at the supercritical minority density. The difference in the computed convection velocities suggests that wave polarization in the resonance zone affects the ratio of the convection velocity with respect to the specific power per particle.

References

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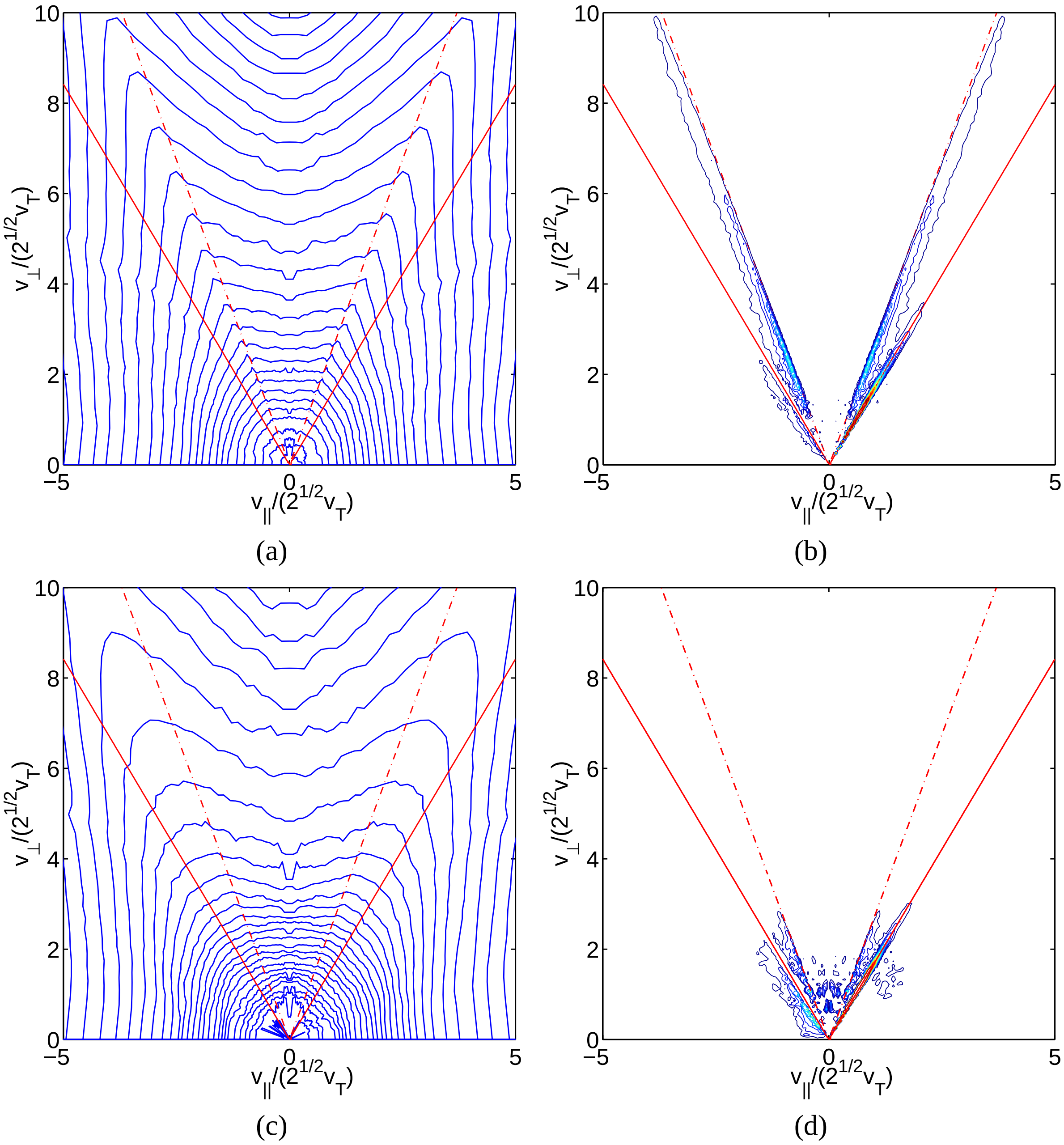

Figure 1. rf heating with (a,b) subcritical and (c,d) supercritical minority density. (a,c) contours of the ion minority distribution function, (b,d) distribution of the convection velocity in velocity space. The trapped-passing boundary is the solid line and the dashed line corresponds to particles with banana tips in the resonance zone.