Computation of equilibrium currents and neoclassical transport for resonant magnetic surfaces in stellarators*

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Introduction

Expressions for bootstrap current in stellarators, which are commonly given in magnetic coordinates, possess resonances at rational magnetic surfaces in the low collisionality regime. In reality, rational magnetic surfaces in stellarators can result in ergodic regions or in regions with magnetic islands. In magnetic coordinates however, a consideration of this effect is not possible. In the present work, the computation of the bootstrap current as well as the Pfirsch-Schlüter current and neoclassical transport are performed for a stellarator magnetic field given in real space coordinates. For this purpose, the technique of integration along magnetic field lines is used. Different regions of the magnetic field are considered, containing rational as well as island magnetic surfaces.

Computational technique

The investigation is performed for resonant regions of the magnetic field with either rational surfaces or island magnetic surfaces. Here, the technique based on the integration along magnetic field lines is used. The method is described in references [1] (parallel current density in stellarators), [2] ($1/\nu$ neoclassical transport in stellarators), and [3] (the plateau regime of neoclassical transport in stellarators). The equations for the transport effects [1-3] in real-space coordinates are always supplemented by the magnetic field line equations as well as by the equations for $\nabla\psi$ where ψ is the magnetic surface label.

The requirement for large integration intervals is one of the main requirements for computations near rational or on island surfaces. This requirement is caused by the fact that for the rotational transform ι (in units of 2π) being close to a rational number, the magnetic field line covers the entire magnetic surface very slowly. To keep the time for the computations in a reasonable range, the computations are made for the simplified stellarator magnetic field with only one toroidal harmonic function containing the associated Legendre function. This case corresponds to the idealized Uragan-3M (U-3M) magnetic field with 9 field periods along the torus. The computations are performed for ι being close to 1/3, 1/2, 2/3 and 3/4. It has to be mentioned that for ι=2/3 and ι=3/4 island surfaces are present.

Computation results for equilibrium currents

It follows from [1] that the average toroidal current density (bootstrap current) is characterized by the dimensionless factor $\lambda_{b1}=\lambda_B+\lambda_{ps}$, where the factors λ_B and λ_{ps} are responsible for the kinetic and hydrodynamic (Pfirsch-Schlüter) parts of the current, respectively. These factors are calculated through integration along magnetic field lines. As it follows from [1], it is convenient to take the point of the global maximum of B on a given field line as the starting point of integration.

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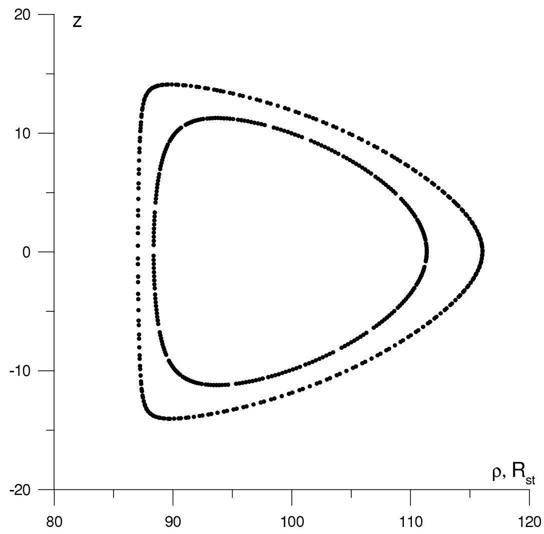

For the given magnetic field, those points lie in one of the magnetic cross-sections with up-down symmetry on the axis of this symmetry where z=0. Fig. 1 shows cross-sections for two magnetic surfaces in an interesting region with different rational values of ι.

Fig. 1.: An inner and outer cross-section of magnetic surfaces corresponding to starting points of integration R_{st}=87.092 and R_{st}=88.41 (for z=0), respectively.

Computational results for λ_{b1} are shown in Fig. 2 as a function of the starting point of integration ($\rho = R_{st}$, z=0). It follows from [1] that for the simplified U-3M configuration the bootstrap current on non-resonant magnetic surfaces is close to that of an equivalent tokamak. This current is shown in Fig. 2 between the resonant regions of the magnetic configuration. It can also be seen from Fig. 2 that for the resonant regions the bootstrap current significantly differs from that on non-resonant regions.

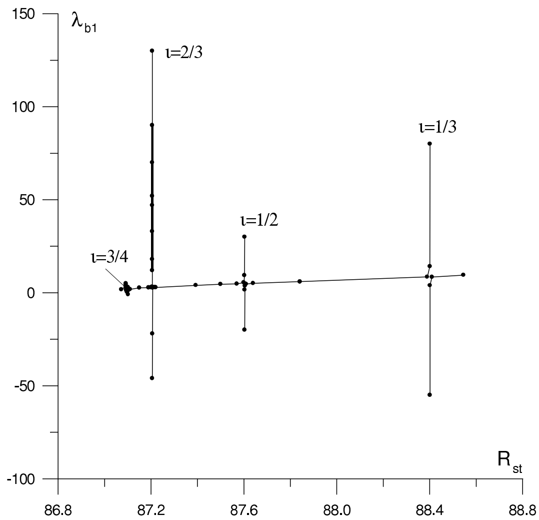

Fig. 2.: The parameter λ_{b1} as a function of the starting point of integration R_{st}. The thick curves correspond to island magnetic surfaces.

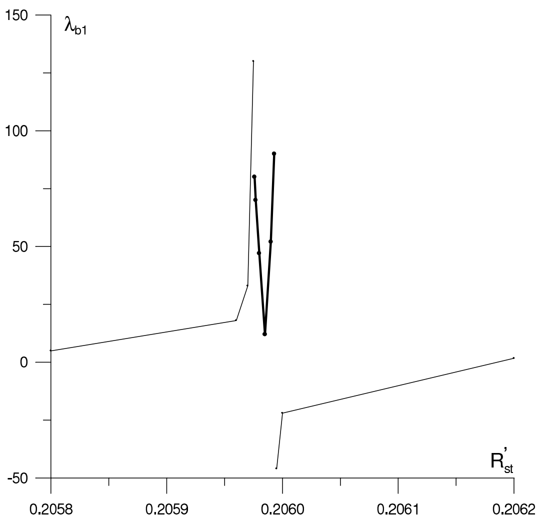

Fig. 3.: The parameter λ_{b1} for ι close to 2/3. The thick curve corresponds to the island magnetic surfaces with R'_{st}=R_{st}-87).

With ι approaching 1/2 or 1/3, the magnitude of λ_{b1} increases. The maximum magnitudes given in Fig. 2 correspond to very small differences of ι from the rational values, $\Delta\iota$. For example, for ι close to 1/2 a ι value of ι=1/2$\pm\Delta\iota$ is used with $\Delta\iota$ $\approx$1/45000. In this case, an integration interval of 5000 turns around the torus is necessary to obtain the correct computational results. For ι values close to 2/3 and to 3/4 island surfaces are found. The λ_{b1} factor is also increased on these surfaces and this increase takes place inside the islands as well as for the non-island surfaces in the close vicinity of these islands. For the islands corresponding to ι=3/4 the increase of λ_{b1} turns out to be smaller than that for the islands at ι=2/3. For ι close to 2/3, an integration interval of 22500 turns around the torus is used in order to reach the necessary accuracy. Fig. 3 shows the results for this region of R_{st}.

The reason for this significant increase of the bootstrap current for ι being close to rational values can be explained as follows. It follows from [1] that the bootstrap current is a result of a displacement of passing particles from the magnetic surface during the particle drift in the inhomogeneous toroidal magnetic field. This drift is periodical in space. The main contribution to the current comes from particles with velocities very close to the trapped-passing boundary in velocity space. For rational magnetic surfaces, particles with small $v_{||}$ remain for a long time in regions of the magnetic field with the same direction of the particle drift across the magnetic surface. This leads to a significant increase of particle displacement from the magnetic surface. Big displacements of passing particles with small $v_{||}$ for ι close to the rational values are confirmed in our computations. Fig. 4 shows the displacement ξ for passing particles with a velocity close to the trapped-passing boundary. The results are for two magnetic surfaces with ι close to 2/3, an island one and a non-island one in the close vicinity of the first surface.

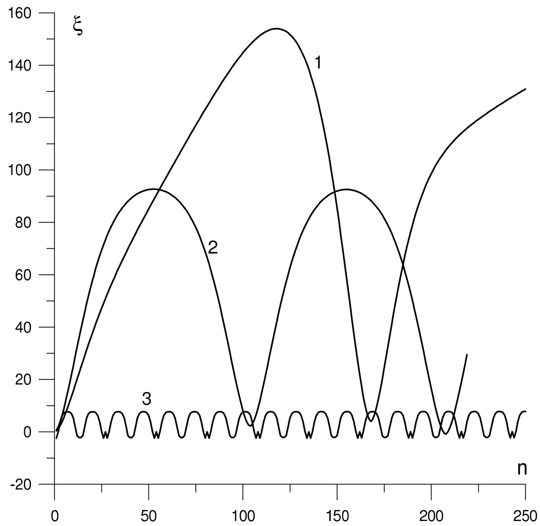

Fig. 4.: Displacement ξ in units of the Larmor radius as a function of the integration length for passing particles close to trapped ones with ι close to 2/3; 1: non-island magnetic surface with R_{st}=87.205975; 2: island magnetic surface with R_{st}=87.205976; 3: short integration interval for a non-island magnetic surface with R_{st}=87.205975; n is the number of integration sub-intervals corresponding to 810 helical field periods for curves 1 and 2 and to 1 helical field period for curve 3, respectively.

For comparison, the distribution of ξ along the magnetic field line is also shown for a small integration interval corresponding to 1/810 of the full interval. It can been seen that the magnitude of the displacement during the short integration interval is significantly smaller than that for the full interval. This magnitude corresponds to the factor λ_{ps}. Note, that the accuracy of the curves for the full integration interval is limited to the amplitude of the curve for the short integration interval. Big values of ξ are observed also for ι close to 1/3 and 1/2. For ι close to 3/4 the ξ values turn out to be essentially smaller than those for ι close to 2/3. For non-resonant magnetic surfaces the ξ values are of the same order as those for short integration intervals. Note also that for resonant magnetic surfaces λ_{ps} (the Pfirsch-Schlüter current is responsible for this quantity) does not differ essentially in magnitude from its value for non-resonant magnetic surfaces.

From the results in Fig. 5 follows that there is no marked difference for $\epsilon_{\text{eff}}^{3/2}$ values computed for non-rational magnetic surfaces and for magnetic surfaces in the narrow neighborhood of the rational surfaces ι=1/3 and ι=1/2. At the same time, $\epsilon_{\text{eff}}^{3/2}$ is somewhat increased for island surfaces corresponding to ι=2/3 and ι=3/4. This increase takes place inside the islands as well as for the non-island surfaces in the vicinity of these islands. The computation of the plateau diffusion coefficient [3] shows that the character of the behavior on resonant and non-resonant magnetic surfaces is approximately the same as for the $\epsilon_{\text{eff}}^{3/2}$ parameter.

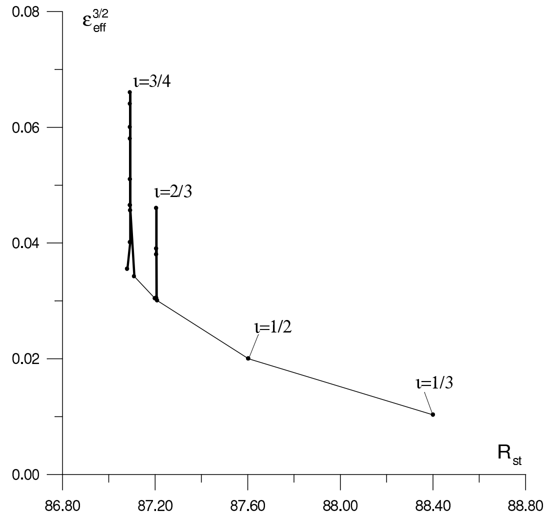

Fig. 5. The parameter $\epsilon_{\text{eff}}^{3/2}$ as a function of R_{st}. The results for ι being close to rational values are given as thick curves.

Summary

The technique [1] used for calculating the parallel current is basically similar to the one proposed in [4]. However, in contrast to [4] our method allows one to perform computations for magnetic fields given in real-space coordinates and to analyze the currents and the transport in island magnetic surfaces which may arise in stellarator magnetic fields for resonant values of the rotational transform ι. Calculations are done for the idealized Uragan-3M magnetic field which satisfies all Maxwell equations up to the numerical accuracy and, therefore, can be realized in a real physical situation. One finds a significant increase of the bootstrap current for ι close to 1/3, 1/2 and 2/3 in contrast to results on non-resonant magnetic surfaces. For these cases one also finds a strong increase of the displacement from the magnetic surface for passing particles with small $v_{||}$.

Note that the resonances of the bootstrap current found in the present computation are valid only in a plasma with a very long mean-free-path. The mean free path must be bigger than the typical field line length on which the direction of the radial drift velocity changes its sign. In addition also the Lamor radius has to be small enough because a finite displacement due to the radial drift leads particles away from the resonant surface. The combination of these two factors leads to the reduction of the resonances of the bootstrap current. There is no marked difference for the Pfirsch-Schlüter current on non-rational magnetic surfaces and on resonant magnetic surfaces. The certain increase of the $1/\nu$ and the plateau transport within island magnetic surfaces is caused by topology change. In this case the coefficient describes the transport within a “local embedded stellarator”.

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