# On the Interaction of a Rotating Magnetic Field with the Plasma in the Kinetic Approximation*

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## Introduction

Recent measurements of the toroidal plasma rotation induced by the Dynamic Ergodic Divertor (DED) at TEXTOR in the $m = 3$, $n = 1$ mode have shown that for magnetic field rotation frequencies at $\pm 1$ kHz and for a static DED field the change in the rotation is always in the direction of plasma current [1] and is independent of the frequency. This feature had not been foreseen during the previous modeling efforts. In the present report a linear kinetic Hamiltonian plasma conductivity model is used to describe the interaction. This provides a better agreement with the experimental results.

## The Model

The tokamak geometry is simplified to a straight periodic radially inhomogeneous cylinder with rotational transform of the magnetic field. In this geometry, the kinetic equation with a Krook collision term is solved using the Hamiltonian formalism (see, e.g.[2]). Due to the axial symmetry of the problem with respect to the Z-axis of the cylindrical coordinates $\mathbf{x} = (r, \vartheta, z)$, canonical action-angle variables can be introduced. The angles $\boldsymbol{\theta} = (\phi, \vartheta_g, z_g)$ are the gyrophase, azimuth and z-coordinate of the guiding center, respectively. The actions $\mathbf{J} = (J_\perp, p_\vartheta, p_z)$ are the perpendicular invariant, $J_\perp \approx m_0 v_\perp^2/(2\omega_c)$, and the covariant components of the generalized momentum, $p_k = (m_0 \dot{\mathbf{r}} + e\mathbf{A}/c)_k$ over $\vartheta$ and $z$ variables. Here, $c$, $e$, $m_0$, and $\omega_c$ are speed of light, particle charge, mass, and cyclotron frequency, respectively. In order to approximate the integral dependence over radius of the current on the electric field by a differential operator, a Larmor radius expansion is introduced. The components of the Larmor radius vector and guiding center coordinates are defined as

$$x^l(\boldsymbol{\theta}, \mathbf{J}) = x_g^l(\boldsymbol{\theta}, \mathbf{J}) + \rho^l(\boldsymbol{\theta}, \mathbf{J}), \qquad \int\limits_{-\pi}^{\pi} \mathrm{d}\phi\, \rho^l(\boldsymbol{\theta}, \mathbf{J}) = 0, \tag{1}$$

where the guiding center coordinates, $\mathbf{x}_g(\boldsymbol{\theta}, \mathbf{J}) = (r_g(\mathbf{J}), \vartheta_g, z_g)$ are independent of the gyrophase. For a single spatial harmonic, $\tilde{E}_l$, $\tilde{j}^l \propto \exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$, where $\mathbf{k} \equiv (0, m_\vartheta, k_z)$ and $\omega$ is the perturbation frequency, the perturbation current density can be written through a differential conductivity operator acting on the perturbation electric field,

$$\tilde{j}^k(r, \vartheta, z) = \frac{1}{r} \sum_{n,n'=0}^{N} (-1)^n \frac{\partial^n}{\partial r^n} \left( r\, \sigma^{kl}_{(n,n')}(r, \mathbf{k})\, \frac{\partial^{n'}}{\partial r^{n'}} \tilde{E}_l(r, \vartheta, z) \right), \tag{2}$$

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where

$$\sigma^{kl}_{(n,n')}(r,\mathbf{k}) = \frac{2\pi i e^2}{r\omega}\sum_{m_\phi=-\infty}^{\infty}\int\limits_0^{\infty\,\mathrm{sgn}(e)}\mathrm{d}J_\perp\int\limits_{-\infty}^{\infty}\mathrm{d}u_\parallel\int\limits_0^{\infty}\mathrm{d}r_0\delta(r-r_g)\frac{\partial\left(p_\vartheta,p_z\right)}{\partial\left(r_0,u_\parallel\right)}\left(a^k_\alpha(n)\right)^*_{\mathbf{m}}$$

$$\times\left(a^l_\beta(n')\right)_{\mathbf{m}}\frac{1}{\mathbf{m}\cdot\boldsymbol{\Omega}-\omega-i\nu}\Omega^\alpha\left((\omega-\mathbf{m}\cdot\boldsymbol{\Omega})\frac{\partial f_0}{\partial J_\beta}+\Omega^\beta\mathbf{m}\cdot\frac{\partial f_0}{\partial\mathbf{J}}\right), \quad (3)$$

$$\left(a^k_\alpha(n)\right)_{\mathbf{m}} = \frac{1}{2\pi n!}\int\limits_{-\pi}^{\pi}\mathrm{d}\phi\,\mathrm{e}^{-im_\phi\phi}\left(\rho^r\right)^n\left(\frac{\partial x^k_g}{\partial\theta^\alpha}\sum_{l=0}^{N-n}\frac{1}{l!}\left(i\mathbf{k}\cdot\boldsymbol{\rho}\right)^l+\frac{\partial\rho^k}{\partial\theta^\alpha}\sum_{l=0}^{N-n-1}\frac{1}{l!}\left(i\mathbf{k}\cdot\boldsymbol{\rho}\right)^l\right),$$

and where the second sum is zero if $N-n<1$. Here $N$ is the expansion order, $\mathbf{k}\cdot\boldsymbol{\rho}=m_\vartheta\rho^\vartheta+k_z\rho^z$, $\mathbf{m}=(m_\phi,m_\theta,k_z)$, $\boldsymbol{\Omega}=(\omega_c,h^\vartheta u_\parallel+v^\theta_E,h^z u_\parallel+v^z_E)$ are the canonical frequencies, $h^i$ and $v^i_E$ are contra-variant components of the unit vector along the unperturbed magnetic field, $\mathbf{h}=\mathbf{B}_0/B_0$ (with $\mathbf{B}_0=\nabla\times\mathbf{A}_0$), and of the electric drift velocity due to the unperturbed electrostatic potential $\Phi_0$, respectively. In (3), the integration variables $r_0$ and $u_\parallel$ are implicitly defined by

$$p_{\vartheta,z} = p_{\vartheta,z}(r_0,u_\parallel)=\left(m_0\mathbf{h}(r_0)u_\parallel+\frac{e}{c}\mathbf{A}_0(r_0)+\frac{m_0\,c}{B_0(r_0)}\mathbf{h}(r_0)\times\nabla\Phi_0(r_0)\right)_{\vartheta,z}. \quad (4)$$

The unperturbed distribution function is taken as an inhomogeneous drifting Maxwellian,

$$f_0=\frac{n_0(r_0)}{\left(2\pi m_0 T_0(r_0)\right)^{3/2}}\exp\left(-\frac{\omega_c(r_0)J_\perp}{T_0(r_0)}-\frac{m_0\left(u_\parallel-V_\parallel(r_0)\right)^2}{2T_0(r_0)}\right), \quad (5)$$

with $n_0$ the equilibrium density, $T_0$ the temperature, and $V_\parallel$ the parallel fluid velocity. Forces acting on plasma can be obtained using the fact that in various frames of reference the total absorbed power given by the integral of $(1/2)\mathrm{Re}(\tilde{j}^k\tilde{E}^*_k)$ over the volume differs only in the mechanical work. Assuming that the moving frame has constant velocity $V^k$, this difference is

$$P_{\mathrm{tot}}-P'_{\mathrm{tot}}=V^k\int\mathrm{d}^3x\sqrt{g}F_k\equiv V^kF^{\mathrm{tot}}_k, \quad (6)$$

where $F^{\mathrm{tot}}_k$ is the covariant component of the total force acting on the plasma ($F^{\mathrm{tot}}_\vartheta\equiv T$ corresponds to a poloidal torque). It follows from the Lorentz transform of the fields and current that the power in the moving frame is $P'_{\mathrm{tot}}=(\omega'/\omega)P_{\mathrm{tot}}$ where $\omega'=\omega-k_jV^j$ is the (Doppler shifted) frequency in the moving frame for the case $V\ll c$. Therefore, the total force is

$$F^{\mathrm{tot}}_j=\frac{k_j}{\omega}P_{\mathrm{tot}}. \quad (7)$$

## Results

For the modeling, the plasma conductivity in the lowest order Larmor radius expansion $N=1$ is used. An orthonormalization method is applied for the numerical solution of the stiff set of first order ordinary differential equations resulting from Maxwell equations. Two cases are considered, the collisionless case with $\nu=0$ and the collisional case where $\nu$ for electrons and ions has been put to $\nu^{e/i}_\perp$ and $\nu^{i/i'}_\perp$ being the transverse diffusion rates for electron-ion and ion-ion collisions, respectively. The modeling is performed for

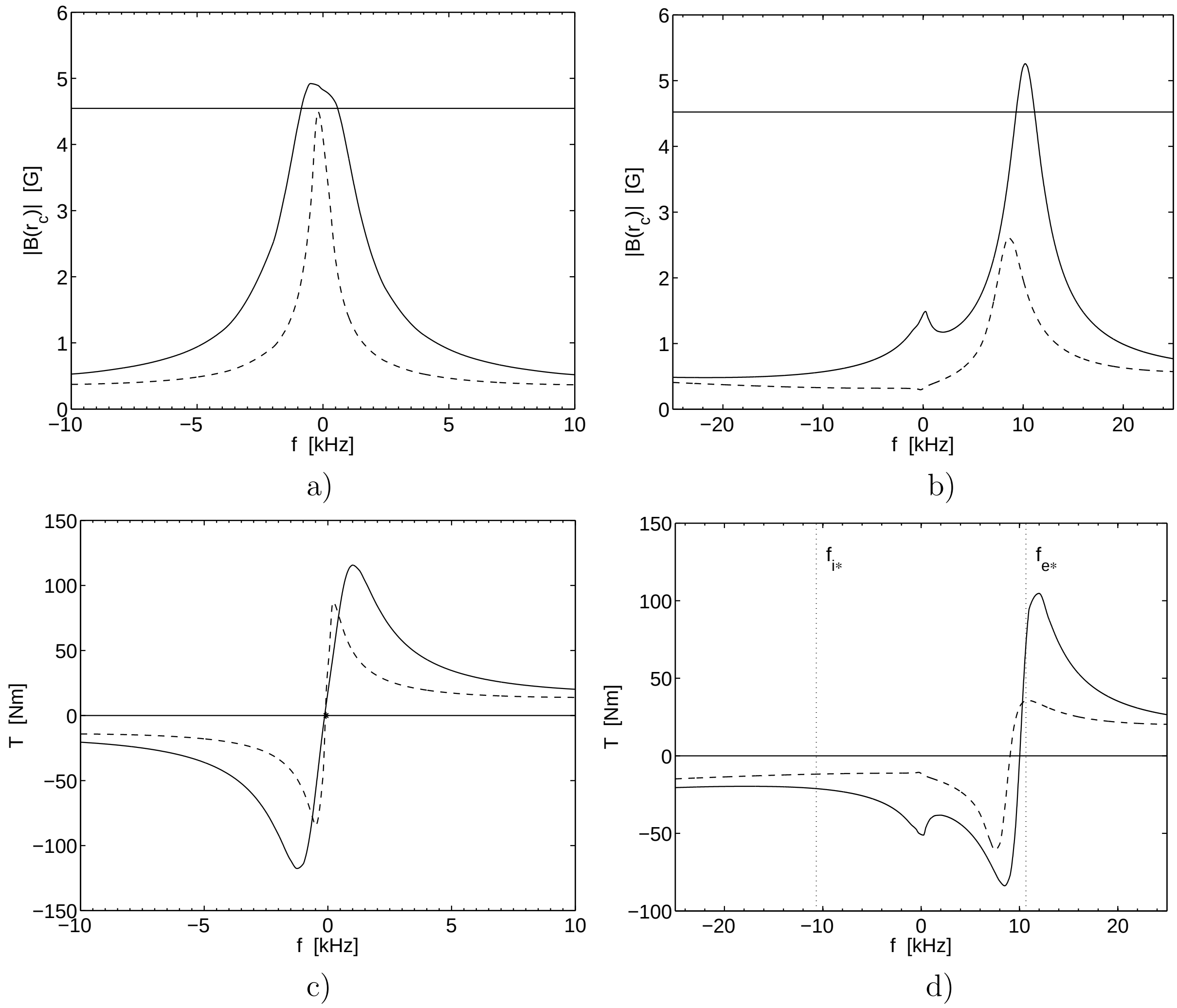


Fig. 1. Left panel without pressure gradients, right panel with pressure gradients. Solid - collisional, dashed - collisionless. Horizontal axis represents the DED frequency $f$.
a, b) Radial magnetic perturbation $\left|\tilde{B}_r(r)\right|$ for $r = 35$ cm. Horizontal line is the ideal MHD value.
c, d) Poloidal torque $T = F_\vartheta^{\rm tot}$. Dotted vertical lines mark ion and electron diamagnetic frequencies.

parameters relevant to TEXTOR-DED. The cylinder period is $L = 2\pi R_0$ where $R_0 = 175$ cm. An ideally conducting wall and perturbation coil are assumed at $r_w = 60$ cm and $r_a = 53$ cm, respectively. The main toroidal magnetic field is $B_0 = 2$ T. The profile of the safety factor $q(r)$ with the location of the rational surface $q = 3$ at $r_{\rm res} = 43.7$ cm has been used. Only the main harmonic of the perturbation current, $m = 12$, $n = 4$ is taken into account. Two cases are considered, the case without pressure gradients (with constant plasma density and temperatures) and the case with pressure gradients with parabolic plasma parameter profiles. Constant (or central) density and temperature values are $n_0 = 1.05 \times 10^{13}$ cm$^{-3}$ and $T_i = T_e = 1.05$ keV, respectively. The radial electric field in the laboratory frame has been put to zero, $\Phi_0 = $ const.

In the case without pressure gradient shown in Fig. 1 a, c, the magnetic field fully penetrates into the plasma at low frequencies. In this case, the radial component, $\tilde{B}_r$, of the static DED field is very close to the result of ideal MHD theory [3] corresponding to $\Delta' = 0$ (see Fig. 1 a). The direction of the torque coincides with the direction of the rotation of the perturbation field and the maximum torque is reached at frequencies around 1 kHz (see Fig. 1 c). This result is in agreement with previous modeling.

In the case with pressure gradients shown in Fig. 1 b, d, the direction of the torque is seen to be in the direction of the diamagnetic current independent of the direction of the DED field rotation as long as the DED frequency does not exceed a certain value, $\omega_r$, which is of the order of the electron diamagnetic frequency $\omega_{*e} = (k_\perp / n_0 \omega_c m_0) \mathrm{d}(n_0 T_e)/\mathrm{d}r$ where $k_\perp = (h_z m_\vartheta - h_\theta k_z)/r$ (see Fig. 1 d). Maximum penetration of the field is now at $\omega = \omega_r$ (see Fig. 1 b). This feature can be explained considering the frequency dependence of the total absorbed power which, following (7), is linked with the torque by the relation $P_{tot} = \omega T / m_\vartheta$. In the simplest collisionless case, the total absorbed power is given by

$$
\begin{aligned}
P_{tot} &= 2\pi^3 m_0^2 \omega L \int\limits_0^\infty \mathrm{d}r_0\, r_0 \int\limits_{-\infty}^{\infty} \mathrm{d}u_\parallel \int\limits_0^{\infty\,\mathrm{sgn}(e)} \mathrm{d}J_\perp \frac{\omega_c}{T_0} f_0 \sum_{\mathbf{m}} \delta\left(\mathbf{m}\cdot\mathbf{\Omega} - \omega\right) |H_{\mathbf{m}}|^2 \qquad (8) \\
&\times \left[\omega'' + \frac{k_\perp}{m_0 \omega_c}\left(\left(\frac{5}{2} - \frac{\omega_c J_\perp}{T_0} - \frac{m_0\left(u_\parallel - V_\parallel\right)^2}{2T_0}\right)\frac{\mathrm{d}T_0}{\mathrm{d}r_0} + m_0\left(u_\parallel - V_\parallel\right)\frac{\mathrm{d}V_\parallel}{\mathrm{d}r_0}\right)\right],
\end{aligned}
$$

where $H_{\mathbf{m}}$ are amplitudes of the Fourier series expansion of the perturbation Hamiltonian, $\tilde{H} = iev^k \tilde{E}_k / \omega$, over canonical angles, and $v^k$ is the particle velocity. The dominant mechanism of absorption is Cerenkov ($m_\phi = 0$) resonance for electrons. Therefore, only the electron contribution to $P_{tot}$ need to be considered. Besides from $\omega$, the sign of $P_{tot}$ is determined by the square brackets where the main contribution to the integral comes from $\omega''$ which is the perturbation frequency in the frame of reference where the given sort of ions is at rest,

$$
\omega'' = \omega - k_\parallel V_\parallel - \omega_* - \omega_E. \qquad (9)
$$

Here, $k_\parallel = h^\vartheta m_\vartheta + h^z k_z \equiv \mathbf{h}\cdot\mathbf{k}$ and $\omega_E$ is the $\mathbf{E}\times\mathbf{B}$ drift frequency which is zero for the specific parameters used in the present computations. The remaining term in square brackets which is proportional to the temperature gradient, and, in principle, is of same order as $\omega''$ (the term with the derivative of $V_\parallel$ is small) provides a 15 % negative shift of the torque reversal frequency, $\omega_r$, away from the electron diamagnetic frequency $\omega_{*e}$. Since the force in the toroidal direction, $F_z^{\mathrm{tot}}$ is expressed through the poloidal torque as $F_z^{\mathrm{tot}} = k_z T / m_\vartheta$ (see (7)), for frequencies in the range $\omega < \omega_r$ the toroidal force is in the direction of plasma current. This force is for low frequencies practically independent of $\omega$ and is finite for a static field. This is in agreement with the experiment [1].

The dependence of the components of the force acting on the plasma on the frequency is such that it tends to bring the electron fluid to be approximately in rest with respect to the rotating field (up do a difference between $\omega_r$ and $\omega_{*e}$). This feature is in a qualitative agreement with long mean-free path drift MHD theory [4] which predicts a resonant behavior and change of the sign of the force at $\omega = \omega_{*e}$. Note that in the range $0 < \omega < \omega_r$ where the direction of the torque is opposite to the direction of the field rotation, the total absorbed power is negative. Therefore, measurements of the active part of DED coil impedance are of interest to verify this theory.

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