INITIAL RESULTS FROM AN INTERNATIONAL COLLABORATION ON NEOCLASSICAL TRANSPORT IN STELLARATORS

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This paper provides an initial status report, summarizing the results obtained to date within the framework of an international collaboration on the topic of neoclassical transport in a number of devices belonging to the stellarator family. The principal goals of this collaboration are:

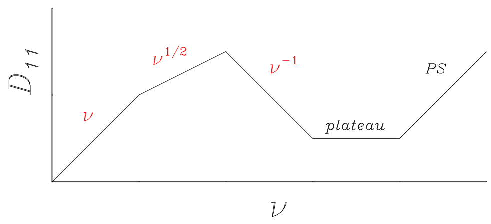
(1) A thorough benchmarking of the various methods used to calculate neoclassical transport coefficients. These include (where appropriate): analytic theory, field-line integration techniques, Monte Carlo simulations, and numerical solutions of the ripple-averaged and drift kinetic equations.
(2) An improved physical understanding of stellarator-specific transport processes.
(3) On the basis of the first two points, the creation of an efficient 'neoclassical data base' to facilitate the analysis of experimental results and to provide an interface for predictive transport codes.

As might be expected, the early stages of the collaboration have concentrated on benchmarking but some aspects of the other two points are also addressed below.

The devices currently under investigation are representative of the extensive configuration space available to stellarators: the classical heliotron/torsatron Large Helical Device (LHD), in operation at Toki, Japan; the heliac TJ-II, in operation at Madrid, Spain; the quasi-axisymmetric National Compact Stellarator Experiment (NCSX), in the planning stage at Princeton, USA; and two advanced stellarators of the Wendelstein line, W7-AS in operation at Garching, Germany, and the helias W7-X which is under construction at Greifswald, Germany.

Basics — The *local* ansatz which underlies neoclassical transport theory allows an ordering of the drift kinetic equation in which the minor-radius and energy coordinates appear only as parameters, reducing a nominally 5D problem to a more manageable 3D. Additionally, it becomes possible to characterize all neoclassical effects in terms of three *mono-energetic* coefficients describing the radial transport, the bootstrap current and the parallel conductivity; the full neoclassical transport matrix is subsequently obtained through the appropriate convolutions of the mono-energetic coefficients with a local Maxwellian. For benchmarking, it is thus sufficient to determine and compare the mono-energetic quantities of interest.

The simplest of all models for the magnetic field strength in a toroidal stellarator may be written as $B/B_0 = 1 - \epsilon_t(r)\cos\theta - \epsilon_h(r)\cos(M\theta - N\phi)$, where (r,θ,ϕ) are the flux coordinates identified with the flux surface, poloidal angle and toroidal angle, respectively, $\epsilon_t = r/R_0$ is the inverse aspect ratio, ϵ_h the magnitude of the stellarator's *helical* ripple, M the multipolarity and N the number of field periods. In the fusion-relevant long-mean-free-path (*lmfp*) regime, particles localized within the stellarator's helical ripples are expected to dominate the radial transport processes. In this regime, where collisions are rare, it is possible to time average the drift kinetic equation over the periodic *bounce* motion of such localized particles to obtain the 2D *bounce-averaged* kinetic equation. Assuming different orderings of the characteristic frequencies appearing in the problem, solutions of the bounce-averaged kinetic equation in several limiting cases have been derived [1]; the scalings of the radial transport coefficient as



a function of collision frequency, ν, obtained from these solutions are indicated in red on the accompanying sketch. Of these stellarator-specific *lmfp* results, $1/\nu$ transport is strongly influenced by the magnetic-field geometry (scaling as $\epsilon_h^{3/2}$) whereas the magnitude of the radial electric field is more decisive in the $\nu^{1/2}$ and ν regimes (which scale as $E^{-3/2}$ and E^{-2}, respectively). The neoclassical theory is less well developed when it comes to the bootstrap current and the parallel conductivity. Results exist for both the collisional and collisionless extremes but are lacking in the range of experimental interest. The effects of the radial electric field are also yet to be investigated.

Benchmarking — To determine neoclassical transport coefficients, one must first solve a kinetic equation and a number of techniques have been developed for doing so. The most general methods are numerical algorithms which deal with the full drift kinetic equation and include a number of schemes based on the Monte Carlo approach [2-5] as well as the DKES (Drift Kinetic Equation Solver) code which employs a variational principle where the solution is expressed using a series of Fourier-Legendre test functions [6]. Both approaches are not without shortcomings (identification of the 'diffusive' and 'convective' contributions to the overall transport when the latter is present in the Monte Carlo simulations; increasingly poor convergence of the DKES results with decreasing collisionality; the considerable computational resources required using either approach), but they can be employed for arbitrarily complex magnetic fields and (usually) at all values of collision frequency.

The ability to handle arbitrarily complex magnetic fields is also a strength of the field-line-integration technique [7]. Here, the drift kinetic equation is considerably simplified by assuming that the fast motion along field lines constitutes the only particle drift *within* a flux surface, ultimately allowing one to express the transport *through* the flux surface as a weighted integral of the geodesic curvature along a field line of 'infinite' length (i.e. sufficiently long to cover the entire magnetic surface). Evaluating the integral numerically, as done in the NEO code [7], provides an efficient means of determining the radial transport for any non-symmetric magnetic configuration in the $1/\nu$ regime. The validity of the results is confined to this regime, however, as a consequence of the initial assumptions.

In contrast, numerical solvers of the *ripple-averaged* kinetic equation may be employed throughout the entire *lmfp* regime. The ripple average is the generalization of the bounce average so as to account for all of phase space (i.e. non-localized particles are treated as well). As realized in the code GSRAKE (General Solution of the Ripple Averaged Kinetic Equation) [8], this allows neoclassical transport coefficients to be determined with great computational efficiency and with excellent convergence properties even at very low collisionalities. The ripple average is not appropriate for arbitrarily complex magnetic fields, however, and its use is restricted to devices which have magnetic fields that can be accurately described within the multiple-helicity model.

An example of benchmarking results is given in figure 1 for the standard configuration of the LHD, a device for which all the methods described above should be applicable. On the left, the mono-energetic radial transport coefficient, normalized to the plateau value of the equivalent axisymmetric tokamak (with circular cross section), Γ_{11}^*, is plotted as a function of the collisionality, ν/v, where v is the velocity of the mono-energetic test particles considered; the flux surface at half the plasma radius has been chosen. Six different values of the radial electric field have been considered, illustrating the strong dependence of Γ_{11}^* on this quantity in the *lmfp* regime. Results from DKES, the DCOM Monte Carlo code [4] and GSRAKE are compared; excellent agreement is obtained with the exception of the most collisionless DCOM result for $E = 0$, due to the large convective transport which appears here in the Monte Carlo simulation. Both positive and negative values of the electric field were considered using DCOM but no statistically significant dependence on the sign of E was found. On the right, the radial dependence of the *effective* helical ripple for $1/\nu$ transport determined using DKES, NEO and an analytic expression [9] is plotted; again the agreement is excellent. By displacing the magnetic flux surfaces towards the high-field side of the device it is possible to realize *drift-optimized* configurations in the LHD. For example, the configuration with a major radius of $R_0 = 3.6$ m (compared to $R_0 = 3.75$ m in the standard case) exhibits significantly reduced $1/\nu$ transport with $\epsilon_{eff} < 0.06$ at the plasma edge. This *inward-shifted* configuration has also been fully benchmarked with success equal to that found for the standard LHD.

A comparison of results for the standard configuration of W7-X is given in figure 2. The upper and lower bounds on the DKES calculations of Γ^*_{11} are indicated for $\nu/v = 3 \times 10^{-6}\ m^{-1}$, illustrating the onset of convergence difficulties for the code. In the plot of ϵ_{eff} as a function of minor radius, a

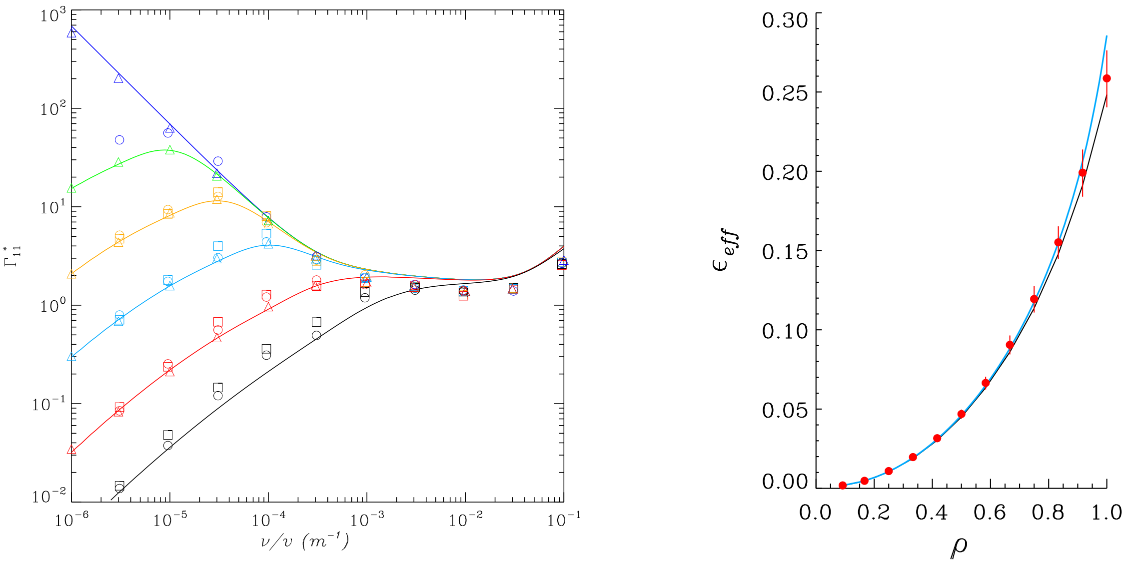


Figure 1. Benchmarking results for the standard configuration of LHD. On the left, the normalized radial transport coefficient is plotted versus the collisionality ν/v (m^{-1}) assuming normalized values of the radial electric field to be $|E|/vB_0 = 3 \times 10^{-3}$, 1×10^{-3}, 3×10^{-4}, 1×10^{-4}, 3×10^{-5} and zero. Numerical results for the $\rho = 0.5$ flux surface are depicted by: triangles for DKES, circles ($E > 0$) and squares ($E < 0$) for the Monte Carlo simulation DCOM, and by the continuous curves for GSRAKE. On the right, the effective helical ripple for $1/\nu$ transport is shown as a function of the normalized minor radius. Analytical results are shown by the black curve, those from NEO in light blue and the DKES calculations are given by red data points with upper and lower bounds indicated by the 'error bars'.

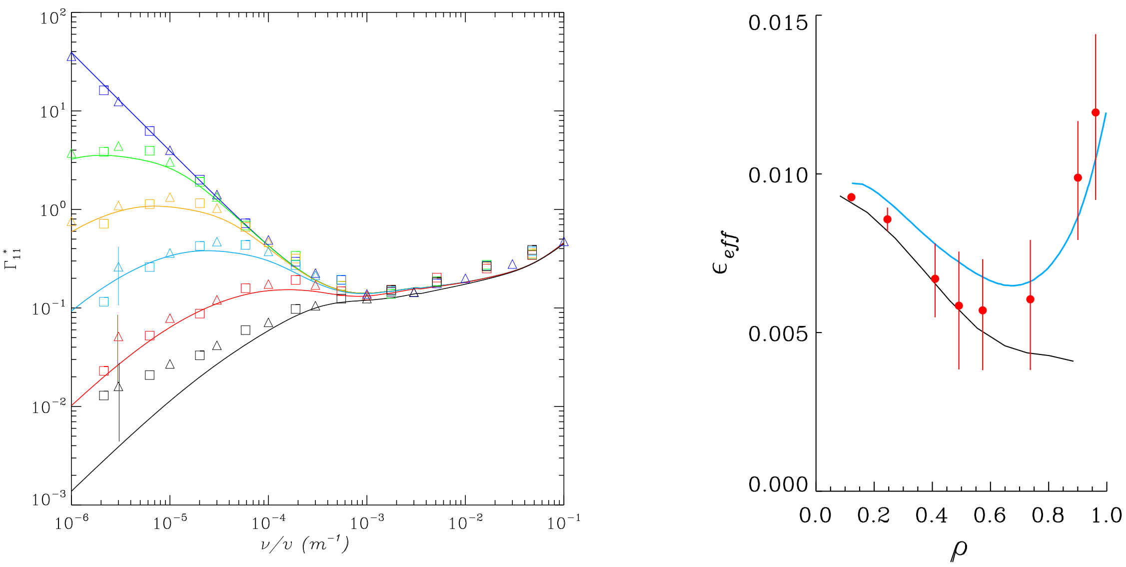


Figure 2. Benchmarking results for the standard configuration of W7-X. The curves and data symbols are as in figure 1.

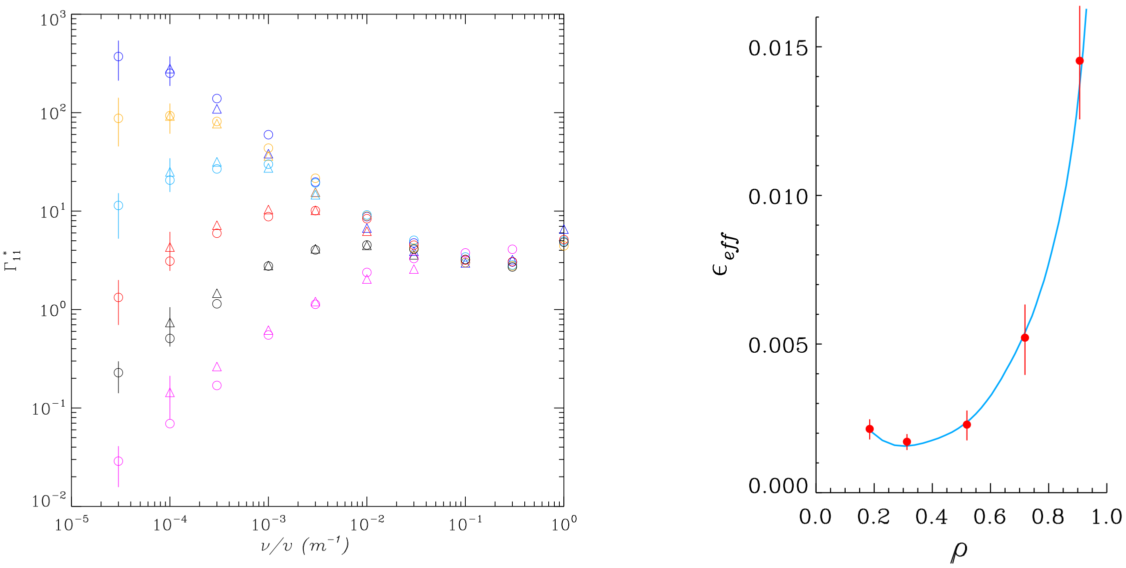


Figure 3. The normalized radial transport coefficient as a function of collisionality for TJ-II (left) and the effective helical ripple as a function of the normalized minor radius for NCSX (right) are shown. The curves and data symbols are as in figure 1; additionally, results are shown for $|E|/vB_0 = 1 \times 10^{-2}$.

discrepancy in the results from DKES and NEO with those of the analytic expression is evident at outer radii. This is due to the higher-order terms in the Fourier decomposition of B (including those due to the discrete coil ripple) which are not accounted for in the analytic theory. The richness of the B spectrum is also responsible for the rather poor convergence of DKES.

This problem is also in evidence for TJ-II (see figure 3), making DKES inappropriate for transport studies of this device deep into the *lmfp* regime. Here one is forced to rely on Monte Carlo simulations; results from the MOCA code [5] have been plotted in figure 3. Also shown is the radial dependence of ϵ_{eff} for NCSX as determined by DKES and NEO.

Outlook — Benchmarking activities will continue with increasing concentration on the bootstrap current. Thus far, only DKES calculations of the bootstrap-current coefficient are available for all configurations and have shown a rather complicated dependence of this quantity on the radial electric field in the *lmfp* regime. This behavior needs to be confirmed and understood. For the radial transport coefficient, the presence of plateau, $1/\nu$ and $\nu^{1/2}$ regimes has been confirmed in all configurations. A ν scaling is also indicated at the lowest collisionalities in some cases. A semi-analytical description of radial transport coefficients (as employed successfully in the analysis of W7-AS experimental data [10]) thus remains a realistic possibility, although some new effects due to specific peculiarities of the magnetic-field structure will need to be accounted for, e.g. the ‘slow’ transition from the $1/\nu$ to $\nu^{1/2}$ scalings observed in strongly drift-optimized configurations as well as modifications to the plateau regime which occur when a significant toroidal mirror term is present in the B spectrum.

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