Stochastic Mapping Technique in Boozer Coordinates*

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Abstract: The Stochastic Mapping Technique (SMT) is an efficient method to solve the Drift Kinetic Equation (DKE) in the long mean free path (LMFP) regime. To overcome the problem of low computational speed, particles in SMT are followed only on Poincaré cuts instead of following stochastic orbits as done in conventional MC methods. For this purpose, various pre-computed maps, e.g., of particle motion between minimum-B cuts, are used.

Up to now the method has been numerically realized for magnetic fields given in real space (cylindrical) coordinates [S.V. Kasilov, W. Kernbichler, V.V. Nemov, M.F. Heyn, Physics of Plasmas 9, 3508 (2002)]. However, the majority of stellarator equilibria are available only in flux coordinates, in particular in Boozer coordinates. In the present report, a numerical realization of SMT in Boozer coordinates and various application of this technique to transport problems in the long mean free path regime are discussed.

1. Introduction

The development of mapping technique has been initiated with the main purpose to describe self-consistently the process of high power rf plasma heating in cases when such process leads to the formation of non-Maxwellian distributions of plasma particles. This changes the wave absorption properties and may influence power deposition profiles, in particular, during ECRH and ECCD. In tokamaks, a problem of finding such a non-Maxwellian distribution function in most cases is successfully solved with help of kinetic equation solvers based on bounce-averaging theory which reduces the dimensionality of the problem to three or two. In stellarators, due to the absence of axial symmetry, the drift kinetic equation has to be solved in the whole phase space which can be handled, in principle, with help of the Monte Carlo methods provided that the efficiency of such a method is high enough. Since in the stochastic mapping technique (SMT) the straightforward integration of test particle drift orbits is replaced by the subsequent mapping of the orbit footprints on the Poincaré cuts which are the surfaces where the magnetic field has a minimum along the field lines (see Ref. [1] for the details), the gain in speed scales with the number of steps needed to integrate the equations of particle motion between the cuts. In case when the stellarator magnetic field is given by interpolation on the spatial grid the gain in speed is two orders of magnitude. If the magnetic field is obtained from the harmonic expansion in flux coordinates, this factor increases significantly. The payment for the speed of the solver is the necessity to pre-calculate and store a significant amount of orbit data which is used in mapping procedure in the form of interpolation that takes 20-30 hours on a PC.

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Originally, stochastic mapping procedure has been developed for the magnetic fields represented in the real space (cylindrical) coordinates. Relatively simple stellarator magnetic field configurations, such as [2], have been considered. However, most stellarator configurations are available only in flux coordinates, in particular, those with finite plasma pressure. Moreover, equilibria with finite pressure which do not assume the existence of embedded flux surfaces needed for flux coordinates are also described in the base coordinate system similar to flux coordinates. Therefore, the extension of SMT to magnetic fields given in flux (Boozer) coordinates was necessary.

2. Peculiarities of mapping in flux coordinates and testing results.

In order to reduce the significant amount of storage of orbit maps to a practically manageable level, already in the real space version of the code a kind of local magnetic coordinate systems has been used. These are coordinates introduced within one toroidal field period so that two of coordinate planes, $x^1 = const$ and $x^2 = const$ contain the magnetic field lines, $\mathbf{B} \cdot \nabla x^1 = \mathbf{B} \cdot \nabla x^2 = 0$, like Clebsch coordinates, and coincide with cylindrical coordinates R and Z on the reference toroidal cut $\varphi = const$. Coordinate systems in neigbouring periods are linked with each other by a magnetic field map, $x^i_{next} = X^i(\mathbf{x}_{prev})$, where functions $X^i(\mathbf{x})$ are given by high order (qubic) interpolation on the grid of starting points $\mathbf{x}$ of the magnetic field line map obtained with help of numerical field line integration from one reference cut to another. As a consequence, maps of particle orbits from one Poincaré cut to another are described in terms of small displacements of the trajectory from the starting field line (they scale with the Larmor radius) which do not require extremely high accuracy needed to separate fast parallel motion from relatively slow perpendicular. Thus, a second order interpolation in space (over $x^{1,2}$ coordinates) is used for the displacements which has to be done for the regions with rather complex boundaries (see Figure 1).

In Boozer coordinates, magnetic field maps become very simple if one defines $x^1 = s$ and $x^2 = \theta_0 \equiv \theta - \iota\varphi$ where (s, θ, φ) and ι are toroidal flux normalized to the edge value, poloidal and toroidal angles, and rotational transform angle, respectively: $x^1_{next} = x^2_{prev}$, $x^2_{next} = x^2_{prev} \pm 2\pi\iota/n$ where n is a number of field periods. At the same time, the region very near the magnetic axis is poorly described by mapping procedure since the assumption of small displacements is violated there for x^2 variable.

Since the dependencies of the orbit map on the test particle momentum module is represented by Taylor expansion, the dimensionality of necessary storage is reduced to three: besides $x^{1,2}$, the dependencies of mapping functions on the pitch, λ, are remaining. This dependencies appear to be rather steep in case of realistic configurations, like W7X (see Fig.2), and are handled using the adaptive grid over λ. Nevertheless, the overall accuracy of interpolation of the maps appears sufficient in order to reproduce trapped collisionless orbits in W7X during many periods of their drift motion. Therefore, in particular, SMT can be used for the estimation of collisionless confinement of fast particles, assuming that orbits in small Larmor radius approximation which follow the contours of parallel adiabatic invariant are representative of fast particles.

Obvious advantage of flux coordinates for local computations of transport coefficients assuming infinitesimal value of Larmor radius can be used also in the mapping procedure: The dimension of the storage can be reduced by one if one is interested in computations just on some particular magnetic surface, and, therefore, the pre-loading time is decreased significantly. Therefore, application of SMT to the computation of transport coefficients in the long mean free path regime can be of interest. First test results of the mono-energetic transport coefficient computations are shown in Fig.3. In particular, collisionality scans with finite the radial electric field agree with corresponding scans in Ref. [3].

References

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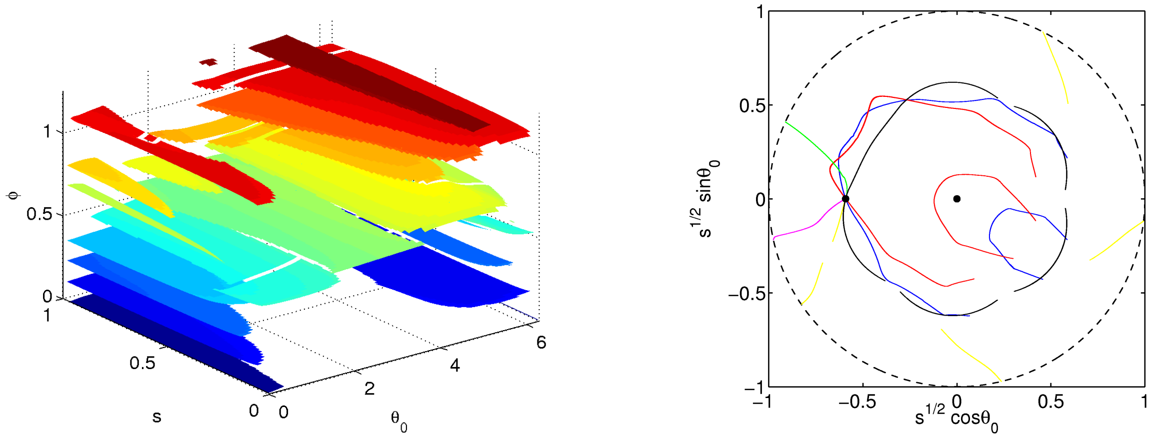

Figure 1: Poincaré cuts in W7-X (left) and collisionless particle orbits in W7-X without radial electric field for various starting values of pitch (right). The magnetic axis and the starting point of orbits are marked by black dots.

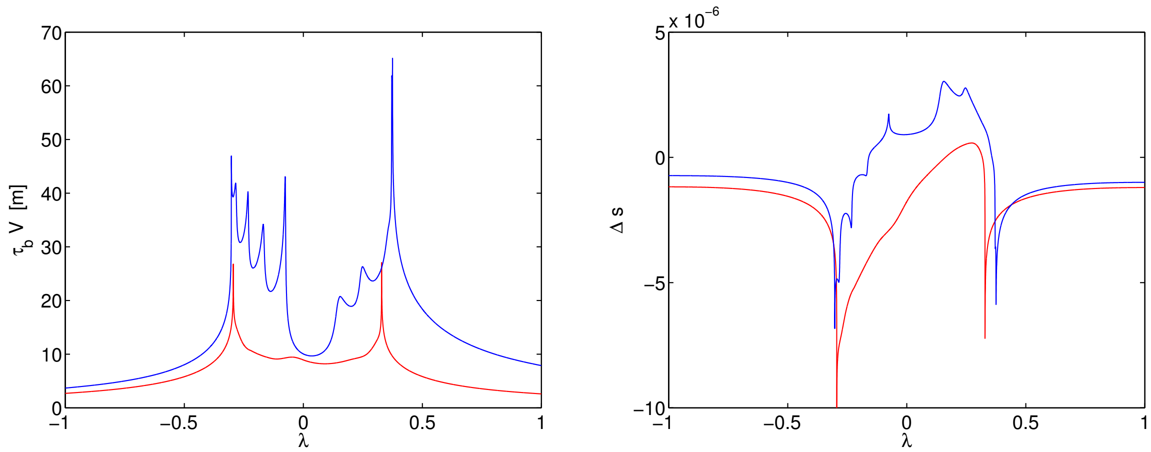

Figure 2: Transition time, τ_b, (left) and displacement over flux label after one transition between Poincaré cuts, Δs, in W7-X (blue) and LHD-360 (red).

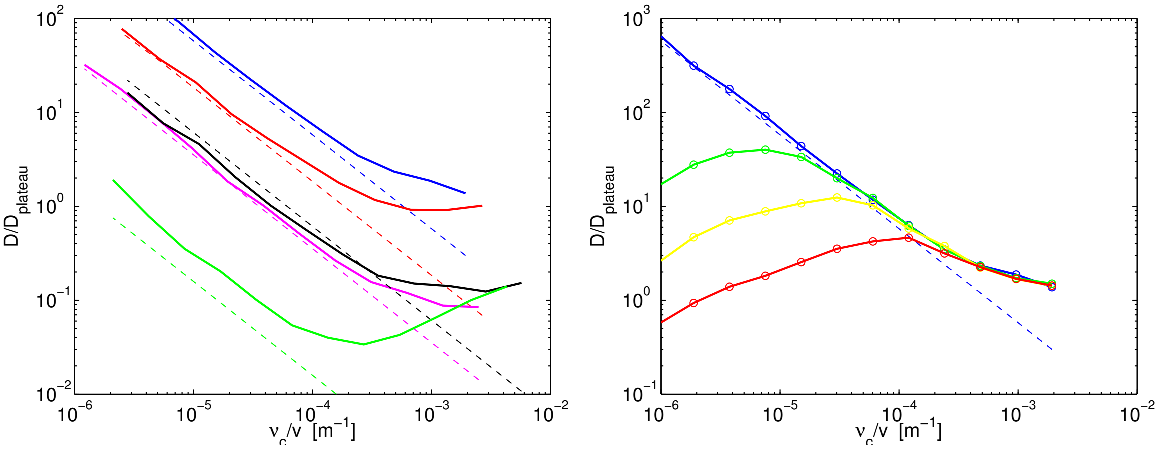

Figure 3: Normalized transport coefficients in the $1/\nu$-regime vs. collisionality for LHD-375 (blue), LHD-360 (red), CHS-qa (black), W7-X (magenta) and NCSX (green) (left). Normalized transport coefficients vs. collisionality for LHD-375 with and without radial electric field, $cE/vB_0 = 0$ (blue), $3 \cdot 10^{-5}$ (green), $1 \cdot 10^{-4}$ (yellow), $3 \cdot 10^{-4}$ (red) (right). Dashed curves in both plots represent normalized transport coefficients computed from $\epsilon_{eff}^{3/2}$ given by the NEO code.